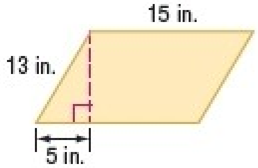
11-1 Areas of Parallelograms and Triangles

Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

1. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h*, of the parallelogram.

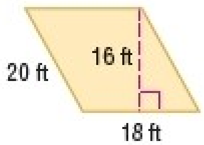
$$a^2 + b^2 = c^2$$
$$5^2 + h^2 = 13^2$$
$$h^2 = 13^2 - 5^2$$
$$h^2 = 169 - 25$$
$$h = \sqrt{144}$$
$$h = 12$$

$$\begin{aligned} A &= bh \\ &= 15(12) \\ &= 180 \end{aligned}$$

$$\begin{aligned} P &= 2(13 + 15) \\ &= 2(28) \\ &= 56 \end{aligned}$$

ANSWER:

56 in., 180 in^2

2. 

SOLUTION:

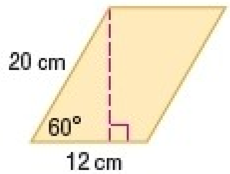
$$\begin{aligned} A &= bh \\ &= 18(20) \\ &= 360 \end{aligned}$$

Each pair of opposite sides of a parallelogram is congruent to each other.

$$\begin{aligned} P &= 2(18+20) \\ &= 2(38) \\ &= 76 \end{aligned}$$

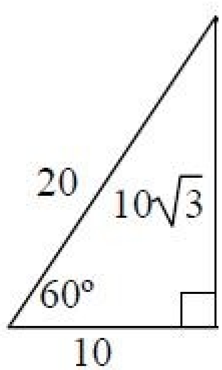
ANSWER:

76 ft, 288 ft^2

3. 

SOLUTION:

Use the 30-60-90 triangle to find the height.



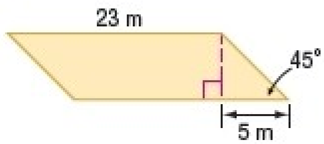
$$\begin{aligned} A &= bh \\ &= 12\left(10\sqrt{3}\right) \\ &\approx 207.85 \end{aligned}$$

Each pair of opposite sides of a parallelogram is congruent to each other.

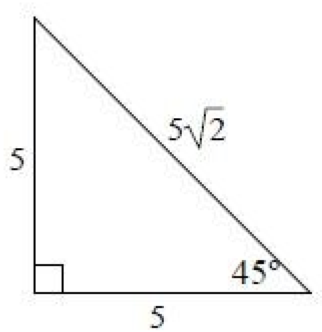
$$\begin{aligned} P &= 2(12 + 20) \\ &= 2(32) \\ &= 64 \end{aligned}$$

ANSWER:

64 cm, 207.8 cm^2

4. 

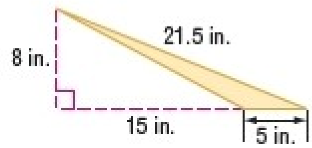
SOLUTION:
Use the 45-45-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 23(5) \\ &= 115 \end{aligned}$$

$$\begin{aligned} P &= 2(23 + 5\sqrt{2}) \\ &= 46 + 10\sqrt{2} \\ &\approx 60.1 \end{aligned}$$

ANSWER:
60.1 m, 115 m^2

5. 

SOLUTION:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(8)(5)$$
$$= 20$$

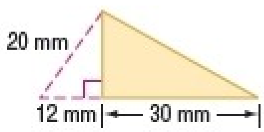
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$a^2 + b^2 = c^2$$
$$8^2 + 15^2 = c^2$$
$$64 + 225 = c^2$$
$$\sqrt{289} = c$$
$$17 = c$$

Therefore, the perimeter is 17 + 5 + 21.5 = 43.5 in.

ANSWER:

43.5 in., 20 in^2

6. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h*, of the triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 12^2 + b^2 &= 20^2 \\ b^2 &= 20^2 - 12^2 \\ b^2 &= 400 - 144 \\ b &= \sqrt{256} \\ b &= 16 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(30)(16) \\ &= 240 \end{aligned}$$

Use the Pythagorean Theorem to find the length of the third side of the triangle.

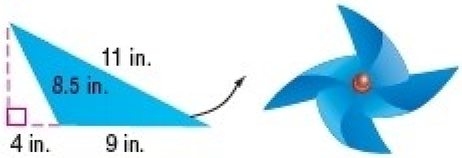
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + 16^2 &= c^2 \\ 900 + 256 &= c^2 \\ \sqrt{1156} &= c \\ 34 &= c \end{aligned}$$

Therefore, the perimeter is 30 + 16 + 34 = 80 mm.

ANSWER:

80 mm, 240 mm^2

7. **CRAFTS** Marquez and Victoria are making pinwheels. Each pinwheel is composed of 4 triangles with the dimensions shown. Find the perimeter and area of one triangle.



SOLUTION:

The perimeter is 9 + 11 + 8.5 or 28.5 in.

Use the Pythagorean Theorem to find the height *h*, of the triangle.

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 8.5^2 - 4^2$$
$$a^2 = 72.25 - 16$$
$$a = 7.5$$

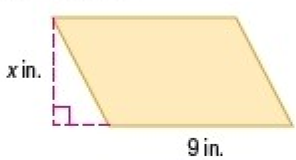
$$\text{Area} = \frac{1}{2}bh$$
$$\text{Area} = \frac{1}{2}(9)(7.5)$$
$$\approx 33.8$$

ANSWER:

28.5 in., 33.8 in^2

Find *x*.

8. $A = 153\ \text{in}^2$



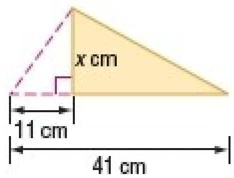
SOLUTION:

$$A = bh$$
$$153 = 9(x)$$
$$\frac{153}{9} = x$$
$$17 = x$$

ANSWER:

17 in.

9. $A = 165 \text{ cm}^2$



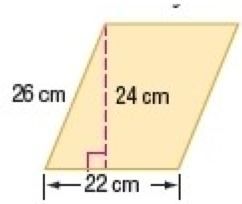
SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}bh \\ 165 &= \frac{1}{2}(30)(x) \\ 165 &= 15x \\ \frac{165}{15} &= x \\ 11 &= x \end{aligned}$$

ANSWER:

11 cm

CCSS STRUCTURE Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

10. 

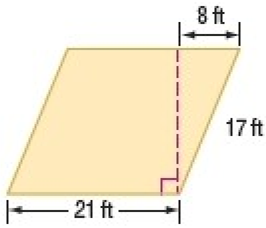
SOLUTION:

$$\begin{aligned} A &= bh \\ &= 22(24) \\ &= 528 \end{aligned}$$

$$\begin{aligned} P &= 2(26+22) \\ &= 2(48) \\ &= 96 \end{aligned}$$

ANSWER:

96 cm, 528 cm^2

11. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h*, of the parallelogram.

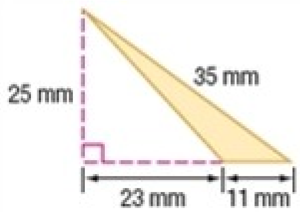
$$a^2 + b^2 = c^2$$
$$8^2 + h^2 = 17^2$$
$$h^2 = 17^2 - 8^2$$
$$h^2 = 289 - 64$$
$$h = \sqrt{225}$$
$$h = 15$$

$$\begin{aligned} A &= bh \\ &= 21(15) \\ &= 315 \end{aligned}$$

$$\begin{aligned} P &= 2(21 + 17) \\ &= 2(38) \\ &= 76 \end{aligned}$$

ANSWER:

76 ft, 315 ft^2

12. 

SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(11)(25) \\ &= 137.5 \end{aligned}$$

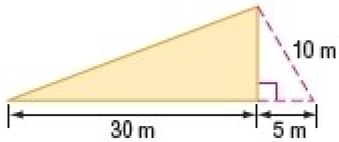
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 25^2 + 23^2 &= c^2 \\ 625 + 529 &= c^2 \\ \sqrt{1154} &= c \\ 34 &\approx c \end{aligned}$$

The perimeter is about 35 + 11 + 34 or 80 mm.

ANSWER:

80 mm, 137.5 mm^2

13. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h* of the triangle.

$$\begin{aligned} a^2+b^2&=c^2\\ 5^2+h^2&=10^2\\ h^2&=10^2-5^2\\ h^2&=100-25\\ h&=\sqrt{75}\\ h&=5\sqrt{3} \end{aligned}$$

$$\begin{aligned} A &= \tfrac{1}{2}bh\\ &= \tfrac{1}{2}(30)\left(5\sqrt{3}\right)\\ &= 75\sqrt{3}\\ &\approx 129.9 \end{aligned}$$

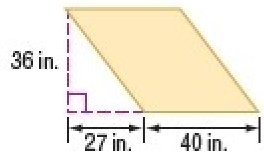
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$\begin{aligned} a^2+b^2&=c^2\\ 30^2+\left(\sqrt{75}\right)^2&=c^2\\ 900+75&=c^2\\ \sqrt{975}&=c\\ 31.2&\approx c \end{aligned}$$

The perimeter is about 8.7 + 30 + 31.2 = 69.9 m.

ANSWER:

69.9 m, 129.9 m^2

14. 

SOLUTION:

$$\begin{aligned} A &= bh \\ &= 40(36) \\ &= 1440 \end{aligned}$$

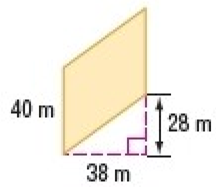
Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 36^2 + 27^2 &= c^2 \\ 1296 + 729 &= c^2 \\ \sqrt{2025} &= c \\ 45 &= c \end{aligned}$$

The perimeter is 2(40 + 45) = 170

ANSWER:

170 in., 1440 in^2

15. 

SOLUTION:

$$\begin{aligned} A &= bh \\ &= 40(38) \\ &= 1520 \end{aligned}$$

Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

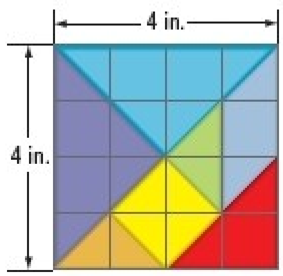
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 38^2 + 28^2 &= c^2 \\ 1444 + 784 &= c^2 \\ \sqrt{2228} &= c \\ 47.2 &\approx c \end{aligned}$$

The perimeter is about 2(40 + 47.2) = 174.4.

ANSWER:

174.4 m, 1520 m^2

16. **TANGRAMS** The tangram shown is a 4-inch square.



a. Find the perimeter and area of the purple triangle. Round to the nearest tenth.
b. Find the perimeter and area of the blue parallelogram. Round to the nearest tenth.

SOLUTION:

a.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(2) \\ &= 4 \end{aligned}$$

Use the Pythagorean Theorem to find the length of the two congruent sides of the triangle. Note that each side is also a hypotenuse for a triangle with sides of length 2.

$$a^2 + b^2 = c^2$$
$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$\sqrt{8} = c$$
$$2.83 \approx c$$

The perimeter is about 2.83 + 2.83 + 4 or 9.7 in.

b.

$$A = bh$$
$$= 2(1)$$
$$= 2$$

Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

$$a^2 + b^2 = c^2$$
$$1^2 + 1^2 = c^2$$
$$2 = c^2$$
$$\sqrt{2} = c$$
$$1.4 \approx c$$

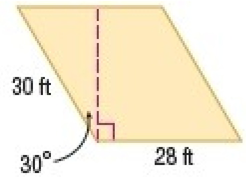
The perimeter is about 2(2 + 1.4) = 6.8.

ANSWER:

a. 9.7 in.; 4 in^2

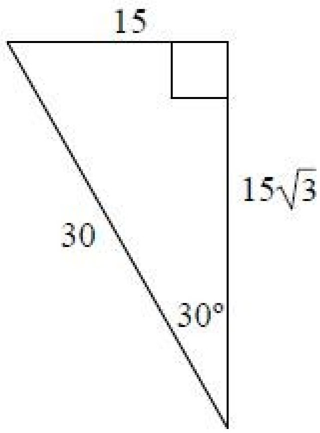
b. 6.8 in.; 2 in^2

CCSS STRUCTURE Find the area of each parallelogram. Round to the nearest tenth if necessary.

17. 

SOLUTION:

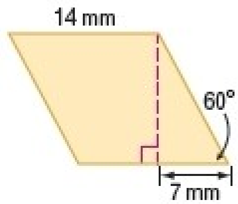
Use the 30-60-90 triangle to find the height.



$$A = bh$$
$$= 28(15\sqrt{3})$$
$$\approx 727.5$$

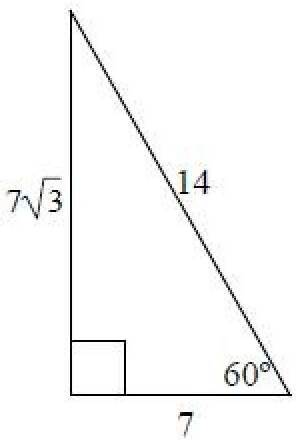
ANSWER:

727.5 ft^2

18. 

SOLUTION:

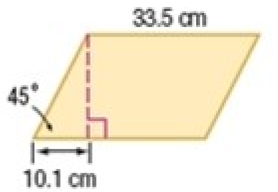
Use the 30-60-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 14\left(7\sqrt{3}\right) \\ &\approx 169.7 \end{aligned}$$

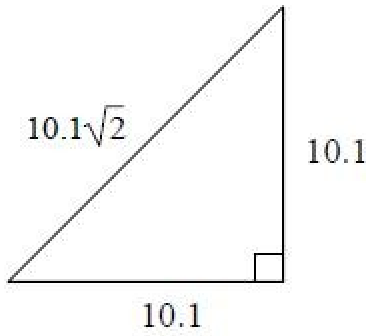
ANSWER:

169.7 mm^2

19. 

SOLUTION:

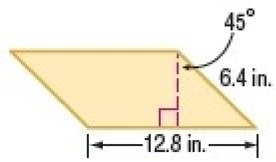
Use the 45-45-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 33.5(10.1) \\ &= 338.4 \end{aligned}$$

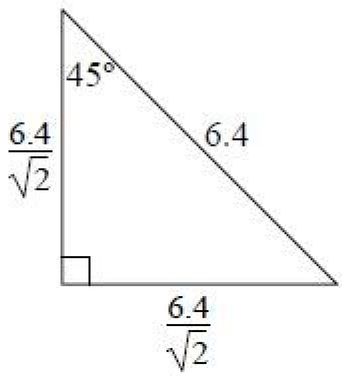
ANSWER:

338.4 cm^2

20. 

SOLUTION:

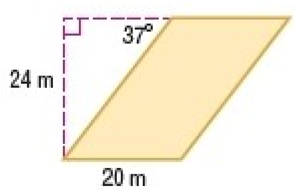
Use the 45-45-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 12.8\left[\frac{6.4}{\sqrt{2}}\right] \\ &\approx 57.9 \end{aligned}$$

ANSWER:

57.9 in^2

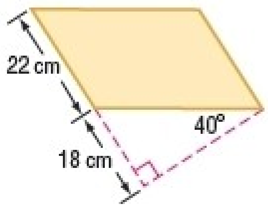
21. 

SOLUTION:

$$\begin{aligned} A &= bh \\ &= 20(24) \\ &= 480 \end{aligned}$$

ANSWER:

480 m^2

22. 

SOLUTION:

Use the tangent ratio of an angle to find the height of the parallelogram.

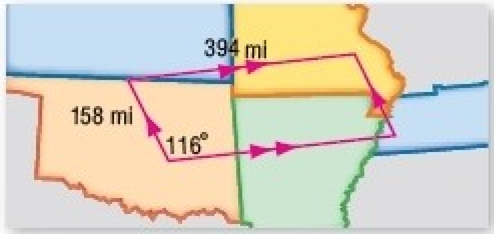
$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 40 = \frac{18}{h}$$
$$h \tan 40 = 18$$
$$h = \frac{18}{\tan 40}$$
$$h \approx 21.45$$

$$A = bh$$
$$= 22\left[\frac{18}{\tan 40}\right]$$
$$\approx 471.9$$

ANSWER:

471.9 cm^2

23. **WEATHER** Tornado watch areas are often shown on weather maps using parallelograms. What is the area of the region affected by the tornado watch shown? Round to the nearest square mile.



SOLUTION:

We have a figure as shown.

Use the cosine ratio of an angle to find the height of the parallelogram.

$$\cos 26° = \frac{h}{158}$$
$$158(\cos 26°) = h$$
$$142 \approx h$$

The area of a parallelogram is the product of its base length b and its height h.

b = 394 and $h \approx 142$. So, the area of the parallelogram is about 394(142) = 55,948.
Therefore, an area of about 55,948 mi^2 is affected by the tornado.

ANSWER:

55,948 mi^2

24. The height of a parallelogram is 4 millimeters more than its base. If the area of the parallelogram is 221 square millimeters, find its base and height.

SOLUTION:

The area of a parallelogram is the product of its base length *b* and its height *h*.

If *b* = the base, then *h* = *b* + 4.

Set up and solve the equation for *b*.

$$\begin{aligned} Area &= bh \\ 221 &= b(b+4) \\ 221 &= b^2 + 4b \\ 0 &= b^2 + 4b - 221 \\ 0 &= (b+17)(b-13) \\ b &= 13 \text{ or } -17 \end{aligned}$$

Length cannot be negative. So, *b* = 13.

Therefore, the base of the parallelogram is 13 mm and the height is 17 mm.

ANSWER:

b = 13 mm; *h* = 17 mm

25. The height of a parallelogram is one fourth of its base. If the area of the parallelogram is 36 square centimeters, find its base and height.

SOLUTION:

The area of a parallelogram is the product of its base length *b* and its height *h*.

Solve the equation for *x*.

$$\begin{aligned} Area &= bh \\ 36 &= b\left(\frac{b}{4}\right) \\ 36 &= \frac{b^2}{4} \\ 144 &= b^2 \\ \pm 12 &= b \end{aligned}$$

Length cannot be negative, so *b* = 12 and *h* = 3.

Therefore, the base of the parallelogram is 12 cm and the height is 3 cm.

ANSWER:

b = 12 cm; *h* = 3 cm

26. The base of a triangle is twice its height. If the area of the triangle is 49 square feet, find its base and height.

SOLUTION:

The area of a triangle is half the product of its base length *b* and its height *h*.

$$\begin{aligned} Area &= \frac{1}{2}bh \\ 49 &= \frac{1}{2}(2h)(h) \\ 49 &= h^2 \\ \pm 7 &= h \end{aligned}$$

Length cannot be negative, so the height is 7 and the base is 14.

ANSWER:

b = 14 ft; h = 7 ft

27. The height of a triangle is 3 meters less than its base. If the area of the triangle is 44 square meters, find its base and height.

SOLUTION:

The area of a triangle is half the product of its base length *b* and its height *h*.

$$\begin{aligned} Area &= \frac{1}{2}bh \\ 44 &= \frac{1}{2}b(b-3) \\ 88 &= b(b-3) \\ 88 &= b^2 - 3b \\ 0 &= b^2 - 3b - 88 \\ 0 &= (b-11)(b+8) \end{aligned}$$

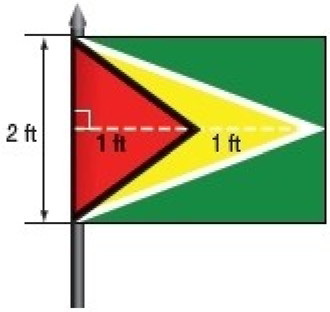
b = 11 or –8. Length cannot be negative, so b = 11.

Therefore, the base of the triangle is 11 m and the height is 8 m.

ANSWER:

b = 11 m; h = 8 m

28. **FLAGS** Omar wants to make a replica of Guyana's national flag.



a. What is the area of the piece of fabric he will need for the red region? for the yellow region?
b. If the fabric costs $3.99 per square yard for each color and he buys exactly the amount of fabric he needs, how much will it cost to make the flag?

SOLUTION:

a. The red region is a triangle with a base of 2 feet and a height of 1 foot. The area of a triangle is half the product of a base *b* and its corresponding height *h*. So, the area of the fabric required for the red region is $\frac{1}{2}(2)(1) = 1 \text{ ft}^2$.

The area of the yellow region is the difference between the areas of the triangle of base 2 ft and height 2 ft and the red region. The area of the triangle of base 2 ft and height 2 ft is $\frac{1}{2}(2)(2) = 2 \text{ ft}^2$ and that of the red region is 1 ft^2.

Therefore, the fabric required for the yellow region is 2 – 1 = 1 ft^2.

b. The amount of fabric that is required for the entire flag is 2 ft · 2 ft or 4 ft^2.

$$3.99 \cdot 4 \text{ ft}^2 = (3 \text{ ft})^2 \cdot x$$
$$15.96 = 9x$$
$$1.77 \approx x$$

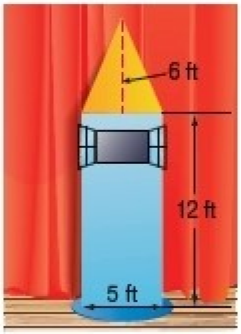
Therefore, the total cost to make the flag will be about $1.77.

ANSWER:

a. $1 \text{ ft}^2; 1 \text{ ft}^2$

b. $1.77

29. **DRAMA** Madison is in charge of the set design for her high school's rendition of *Romeo and Juliet*. One pint of paint covers 80 square feet. How many pints will she need of each color if the roof and tower each need 3 coats of paint?



SOLUTION:

The area of the roof is equal to 2.5 ft · 6 ft or 15 ft^2. If 3 coats of paint are needed, 15 · 3 or 45 square feet need to be painted.

One pint of yellow paint can cover 80 square feet, so only 1 pint of yellow paint is needed.

The area of the tower is equal to 12 ft · 5 ft or 60 ft^2. If 3 coats of paint are needed, 60 · 3 or 180 square feet need to be painted.

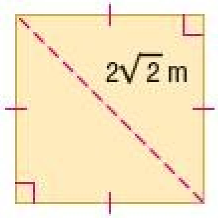
$$180 = 80x$$
$$2.25 = x$$

2.25 pints of blue paint will be needed, so she will need to purchase 3 pints.

ANSWER:

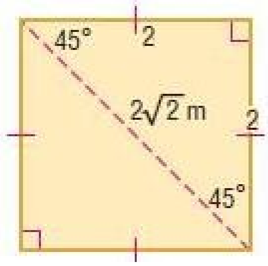
1 pint of yellow, 3 pints of blue

Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.

30. 

SOLUTION:

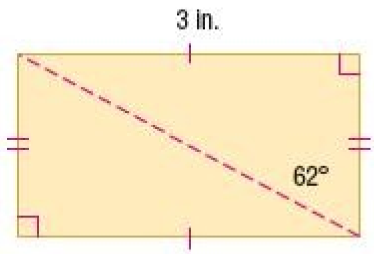
Use the 45°-45°-90° triangle to find the lengths of the sides.



The perimeter is 4 • 2 = 8
The area is 2^2 or 4 m^2.

ANSWER:

8 m; 4 m^2

31. 

SOLUTION:

Use trigonometry to find the width of the rectangle.

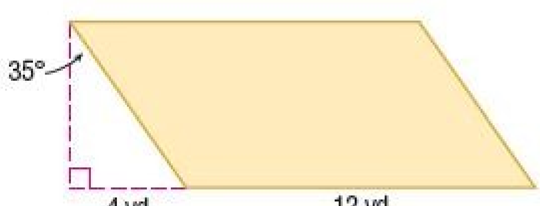
$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 62 = \frac{3}{w}$$
$$w\tan 62 = 3$$
$$w = \frac{3}{\tan 62}$$
$$w \approx 1.6$$

$$P = 2\left(3 + \frac{3}{\tan 62}\right)$$
$$= 6 + \frac{6}{\tan 62}$$
$$\approx 9.19$$

$$A = lw$$
$$= 3 \cdot \frac{3}{\tan 62}$$
$$\approx 4.79$$

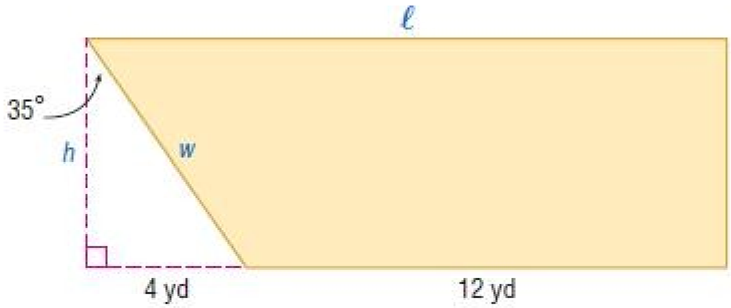
ANSWER:

9.19 in.; 4.79 in^2

32. 

SOLUTION:

Use trigonometry to find the side length of the parallelogram.



$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 35 = \frac{4}{w}$$
$$w\sin 35 = 4$$
$$w = \frac{4}{\sin 35}$$
$$w \approx 7.0$$

$$P = 2\left(12 + \frac{4}{\sin 35}\right)$$
$$= 24 + \frac{8}{\sin 35}$$
$$\approx 37.95$$

Use trigonometry to find h.

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 35 = \frac{4}{h}$$
$$h\tan 35 = 4$$
$$h = \frac{4}{\tan 35}$$
$$h \approx 5.7$$

$$A = lh$$
$$= 12\left(\frac{4}{\tan 35}\right)$$
$$\approx 68.55$$

ANSWER:

37.95 yd; 68.55 yd^2

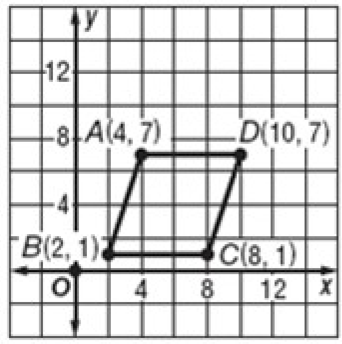
11-1 Areas of Parallelograms and Triangles

COORDINATE GEOMETRY Find the area of each figure. Explain the method that you used.

33. ▱*ABCD* with *A*(4, 7), *B*(2, 1), *C*(8, 1), and *D*(10, 7)

SOLUTION:

Graph the diagram.



The base of the parallelogram is horizontal and from $x = 2$ to $x = 8$, so it is 6 units long.

The height of the parallelogram vertical and from $y = 1$ to $y = 7$, so it is 6 units long.

The area is (6)(6) = 36 units^2.

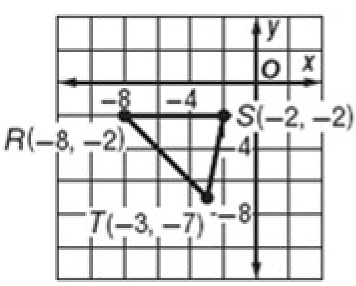
ANSWER:

36 units^2; Graph the parallelogram, then measure the length of the base and the height and calculate the area.

34. Δ*RST* with *R*(–8, –2), *S*(–2, –2), and *T*(–3, –7)

SOLUTION:

Graph the diagram.



The base of the triangle is horizontal and from $x = –2$ to $x = –8$, so it is 6 units long.

The height of the triangle vertical and from $y = –2$ to $y = –7$, so it is 5 units long.

The area is 0.5(6)(5) = 15 units^2.

ANSWER:

15 units^2; Graph the triangle, then measure the length of the base and the height and calculate the area.

35. **HERON'S FORMULA** Heron's Formula relates the lengths of the sides of a triangle to the area of the triangle. The formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the *semiperimeter*, or one half the perimeter, of the triangle and a, b, and c are the side lengths.

a. Use Heron's Formula to find the area of a triangle with side lengths 7, 10, and 4.

b. Show that the areas found for a 5–12–13 right triangle are the same using Heron's Formula and using the triangle area formula you learned earlier in this lesson.

SOLUTION:

a. Substitute a = 7, b = 10 and c = 4 in the formula.

$$s = \frac{a+b+c}{2} = \frac{7+10+4}{2} = 10.5$$
$$A = \sqrt{10.5(10.5-7)(10.5-10)(10.5-4)}$$
$$= \sqrt{119.4375}$$
$$\approx 10.9 \text{ unit}^2$$

b.

$$\sqrt{s(s-a)(s-b)(s-c)} \stackrel{?}{=} \frac{1}{2}bh$$
$$\sqrt{15(15-5)(15-12)(15-13)} \stackrel{?}{=} \frac{1}{2}(5)(12)$$
$$\sqrt{15(10)(3)(2)} \stackrel{?}{=} 30$$
$$\sqrt{900} \stackrel{?}{=} 30$$
$$30 = 30 \checkmark$$

ANSWER:

a. 10.9 units^2

b.

$$\sqrt{s(s-a)(s-b)(s-c)} \stackrel{?}{=} \frac{1}{2}bh$$
$$\sqrt{15(15-5)(15-12)(15-13)} \stackrel{?}{=} \frac{1}{2}(5)(12)$$
$$\sqrt{15(10)(3)(2)} \stackrel{?}{=} 30$$
$$\sqrt{900} \stackrel{?}{=} 30$$
$$30 = 30$$

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the area and perimeter of a rectangle.

a. ALGEBRAIC A rectangle has a perimeter of 12 units. If the length of the rectangle is x and the width of the rectangle is y, write equations for the perimeter and area of the rectangle.

b. TABULAR Tabulate all possible whole-number values for the length and width of the rectangle, and find the area for each pair.

c. GRAPHICAL Graph the area of the rectangle with respect to its length.

d. VERBAL Describe how the area of the rectangle changes as its length changes.
e. ANALYTICAL For what whole-number values of length and width will the area be greatest? least? Explain your reasoning.

SOLUTION:
a. $P = 2x + 2y$; $A = xy$
b. Solve for y.

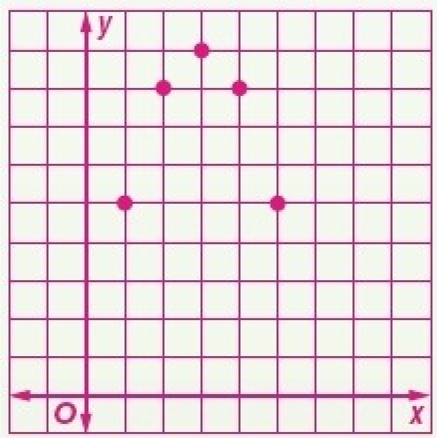
$$\begin{aligned} 12 &= 2x + 2y \\ 12 - 2x &= 2y \\ 6 - x &= y \end{aligned}$$

Now input values for x to get corresponding values for y.

Find xy to get the area.

Length, x	Width, y	Area
1	5	5
2	4	8
3	3	9
4	2	8
5	1	5

c. Length is x and is inputted as the x-coordinates. Area is inputted as the y-coordinates.



d. Sample answer: The plots (and the y-coordinates) go up as the x-coordinates move from 1 to 3, and then move down as the x-coordinates move from 3 to 5.

The area increases as the length increases from 1 to 3, is highest at 3, then decreases as the length increases to 5.

e. Sample answer: The graph reaches its highest point when $x = 3$, so the area of the rectangle will be greatest when the length is 3. The graph reaches its lowest points when $x = 1$ and 5, so the area of the rectangle will be the smallest when the length is 1 or 5, assuming the lengths are always whole numbers.

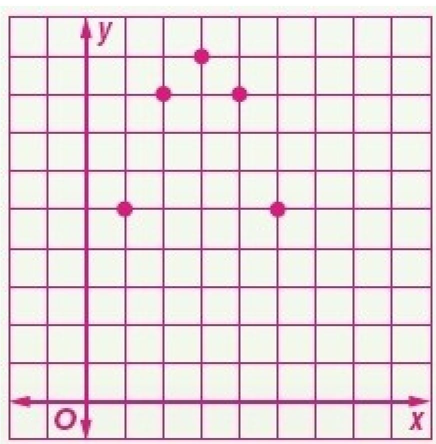
ANSWER:

a. $P = 2x + 2y$; $A = xy$

b.

Length, x	Width, y	Area
1	5	5
2	4	8
3	3	9
4	2	8
5	1	5

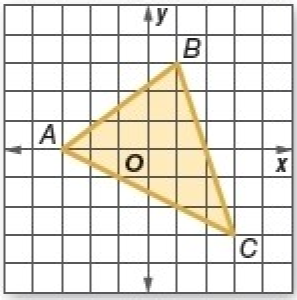
c.



d. Sample answer: The area increases as the length increases from 1 to 3, is highest at 3, then decreases as the length increases to 5.

e. Sample answer: The graph reaches its highest point when $x = 3$, so the area of the rectangle will be greatest when the length is 3. The graph reaches its lowest points when $x = 1$ and 5, so the area of the rectangle will be the smallest when the length is 1 or 5.

37. **CHALLENGE** Find the area of ΔABC. Explain your method.

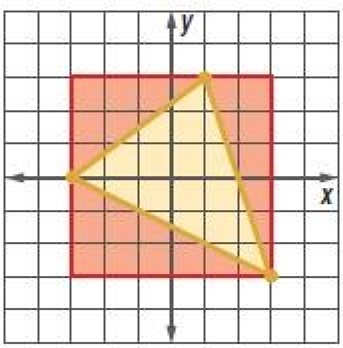


SOLUTION:

One method of solving would be to find the length of one of the bases and then calculate the corresponding height. The length of the sides can be found by using the distance formula; for example by finding the distance between A and C to calculate the length of AC. The height will be more challenging, because we will need to determine the perpendicular from point B to line AC.

We could also use Heron's formula, which is described in problem #35 in this lesson.

A third method will be to inscribe the triangle in a 6 by 6 square. When we do this, we form three new triangles as shown.



The area of the square is 36 units^2. The three new triangles are all right triangles and the lengths of their sides can be found by subtracting the coordinates.

The areas of the three triangles are 6 unit^2, 6 unit^2, and 9 unit^2 respectively. Therefore, the area of the main triangle is the difference or 15 unit^2.

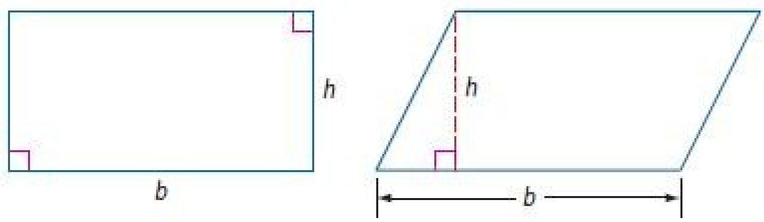
ANSWER:

15 units^2; Sample answer: I inscribed the triangle in a 6-by-6 square. I found the area of the square and subtracted the areas of the three right triangles inside the square that were positioned around the given triangle. The area of the given triangle is the difference, or 15 units^2.

38. **CCSS ARGUMENTS** Will the perimeter of a nonrectangular parallelogram *sometimes*, *always*, or *never* be greater than the perimeter of a rectangle with the same area and the same height? Explain.

SOLUTION:

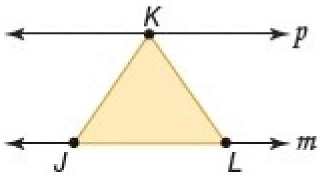
Always; sample answer: If the areas are equal, the perimeter of the nonrectangular parallelogram will always be greater because the length of the side that is not perpendicular to the height forms a right triangle with the height. The height is a leg of the triangle and the side of the parallelogram is the hypotenuse of the triangle. Since the hypotenuse is always the longest side of a right triangle, the non-perpendicular side of the parallelogram is always greater than the height. The bases of the quadrilaterals have to be the same because the areas and the heights are the same. Since the bases are the same and the height of the rectangle is also the length of a side, the perimeter of the parallelogram will always be greater.



ANSWER:

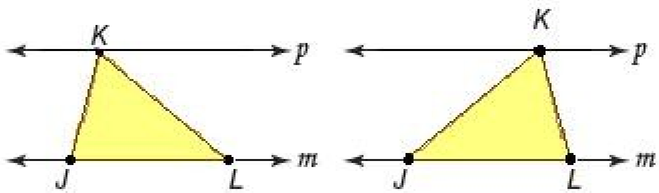
Always; sample answer: If the areas are equal, the perimeter of the nonrectangular parallelogram will always be greater because the length of the side that is not perpendicular to the height forms a right triangle with the height. The height is a leg of the triangle and the side of the parallelogram is the hypotenuse of the triangle. Since the hypotenuse is always the longest side of a right triangle, the non-perpendicular side of the parallelogram is always greater than the height. The bases of the quadrilaterals have to be the same because the areas and the heights are the same. Since the bases are the same and the height of the rectangle is also the length of a side, the perimeter of the parallelogram will always be greater.

39. **WRITING IN MATH** Points *J* and *L* lie on line *m*. Point *K* lies on line p. If lines *m* and p are parallel, describe how the area of ΔJKL will change as *K* moves along line p.



SOLUTION:

Sample answer: The area will not change as *K* moves along line p. Since lines *m* and p are parallel, the perpendicular distance between them is constant. That means that no matter where *K* is on line p, the perpendicular distance to line p, or the height of the triangle, is always the same.



Since point *J* and *L* are not moving, the distance between them, or the length of the base, is constant. Since the height of the triangle and the base of the triangle are both constant, the area will always be the same.

ANSWER:

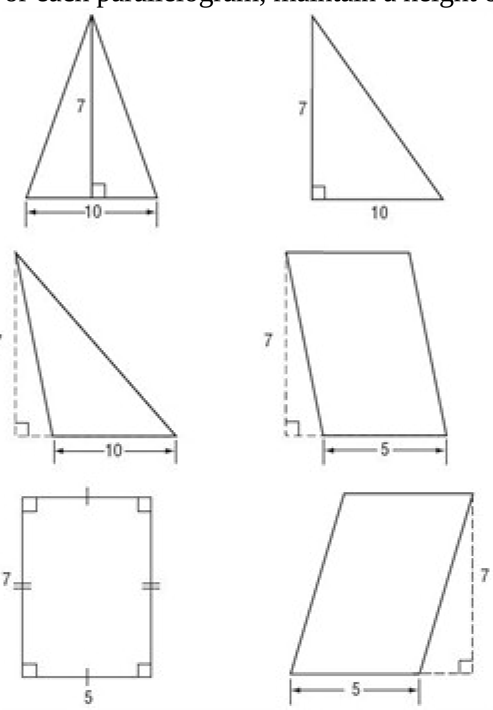
Sample answer: The area will not change as *K* moves along line p. Since lines *m* and p are parallel, the perpendicular distance between them is constant. That means that no matter where *K* is on line p, the perpendicular distance to line p, or the height of the triangle, is always the same. Since point *J* and *L* are not moving, the distance between them, or the length of the base, is constant. Since the height of the triangle and the base of the triangle are both constant, the area will always be the same.

40. **OPEN ENDED** The area of a polygon is 35 square units. The height is 7 units. Draw three different triangles and three different parallelograms that meet these requirements. Label the base and height on each.

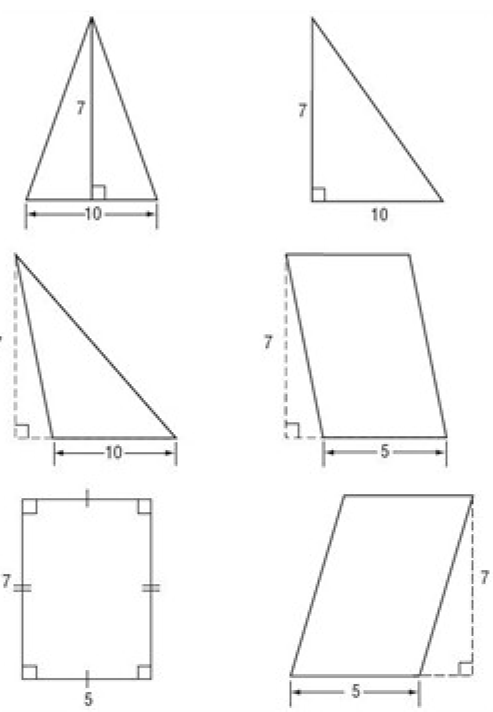
SOLUTION:

For each triangle, maintain a height of 7 and a base of 10, and the area will not change from 35 square units.

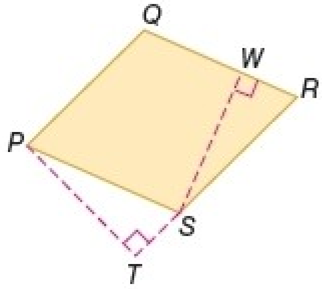
For each parallelogram, maintain a height of 7 and a base of 5, and the area will not change from 35 square units.



ANSWER:



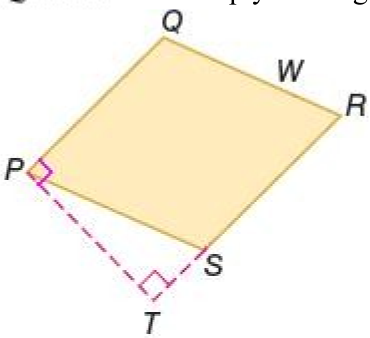
41. **WRITING IN MATH** Describe two different ways you could use measurement to find the area of parallelogram *PQRS*.



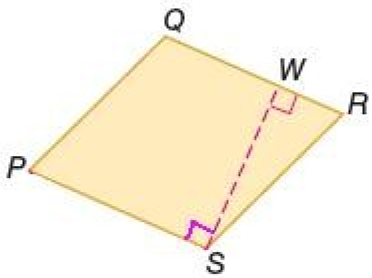
SOLUTION:

Sample answer:
The area of a parallelogram is the product of the base and the height, where the height is perpendicular to the base. Opposite sides of a parallelogram are parallel so either base can be multiplied by the height to find the area.

To find the area of the parallelogram, you can measure the height $\overline{PT}$ and then measure one of the bases $\overline{PQ}$ or $\overline{SR}$ and multiply the height by the base to get the area.



You can also measure the height $\overline{SW}$ and measure one of the bases $\overline{QR}$ or $\overline{PS}$ and then multiply the height by the base to get the area.

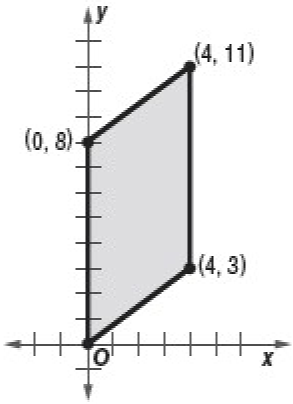


It doesn't matter which side you choose to use as the base, as long as you use the height that is perpendicular to that base to calculate the area.

ANSWER:

Sample answer: To find the area of the parallelogram, you can measure the height $\overline{PT}$ and then measure one of the bases $\overline{PQ}$ or $\overline{SR}$ and multiply the height by the base to get the area. You can also measure the height $\overline{SW}$ and measure one of the bases $\overline{QR}$ or $\overline{PS}$ and then multiply the height by the base to get the area. It doesn't matter which side you choose to use as the base, as long as you use the height that is perpendicular to that base to calculate the area.

42. What is the area, in square units, of the parallelogram shown?



A 12
B 20
C 32
D 40

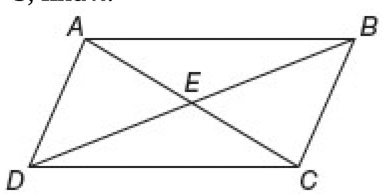
SOLUTION:
One of the bases along the y-axis from $y = 0$ to $y = 8$, so it is 8 units.

The height is horizontal and from $x = 0$ to $x = 4$, so the height is 4 units.

The area is $(8)(4) = 32$ units^2. The correct choice is C.

ANSWER:
C

43. **GRIDDED RESPONSE** In parallelogram $ABCD$, $\overline{BD}$ and $\overline{AC}$ intersect at E. If $AE = 9$, $BE = 3x - 7$, and $DE = x + 5$, find x.



SOLUTION:
In a parallelogram, diagonals bisect each other. So, $BE = DE$, and $3x - 7 = x + 5$.

$$3x - 7 = x + 5$$
$$2x - 7 = 5$$
$$2x = 12$$
$$x = 6$$

ANSWER:
6

44. A wheelchair ramp is built that is 20 inches high and has a length of 12 feet as shown. What is the measure of the angle x that the ramp makes with the ground, to the *nearest* degree?

F 8
G 16
H 37
J 53

SOLUTION:

Use the sine ratio of the angle $x°$ to find the value. Here opposite side is 20 inches and the hypotenuse is 12 ft = 144 in.

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin x = \frac{20}{144}$$
$$x = \sin^{-1}\left(\frac{20}{144}\right)$$
$$x \approx 8°$$

Therefore, the correct choice is F.

ANSWER:

F

45. **SAT/ACT** The formula for converting a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius. Which of the following is the Celsius equivalent to a temperature of 86° Fahrenheit?

A 15.7° C
B 30° C
C 65.5° C
D 122.8° C
D 186.8° C

SOLUTION:

Substitute 86 for F in the equation and solve for C.

$$86 = \frac{9}{5}C + 32$$
$$54 = \frac{9}{5}C$$
$$54\left(\frac{5}{9}\right) = C$$
$$30 = C$$

Therefore, the correct choice is B.

ANSWER:

B

Write the equation of each circle.

46. center at origin, $r = 3$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

$(h, k) = (0, 0)$ and $r = 3$. Therefore, the equation is $(x - 0)^2 + (y - 0)^2 = 3^2$ or $x^2 + y^2 = 9$.

ANSWER:

$x^2 + y^2 = 9$

47. center at origin, $d = 12$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

$(h, k) = (0, 0)$ and $r = d \div 2 = 6$. Therefore, the equation is $(x - 0)^2 + (y - 0)^2 = 6^2$. or $x^2 + y^2 = 36$.

ANSWER:

$x^2 + y^2 = 36$

48. center at (–3, –10), $d = 24$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

$(h, k) = (-3, -10)$ and $r = d \div 2 = 12$. Therefore, the equation is $(x - [-3])^2 + (y - [-10])^2 = r^2$ or $(x + 3)^2 + (y + 10)^2 = 144$.

ANSWER:

$$(x+3)^2 + (y+10)^2 = 144$$

49. center at (1, –4), $r = \sqrt{17}$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

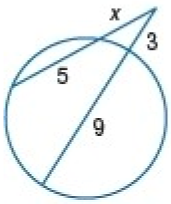
$(h, k) = (1, -4)$ and $r = \sqrt{17}$.

Therefore, the equation is $(x - 1)^2 + (y - [-4])^2 = \left(\sqrt{17}\right)^2$ or $(x - 1)^2 + (y + 4)^2 = 17$.

ANSWER:

$$(x-1)^2 + (y+4)^2 = 17$$

Find *x* to the nearest tenth. Assume that segments that appear to be tangent are tangent.

50. 

SOLUTION:

If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

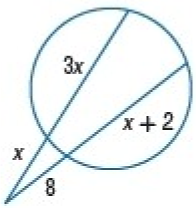
$$\begin{aligned} x(x+5) &= 3(12) \\ x^2+5x &= 36 \\ x^2+5x-36 &= 0 \\ (x+9)(x-4) &= 0 \end{aligned}$$

$x = 4$ or -9

Since x is a length, it cannot be negative. Therefore, $x = 4$.

ANSWER:

4

51. 

SOLUTION:

If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

$$x(4x) = 8(x+10)$$
$$4x^2 = 8x + 80$$
$$4x^2 - 8x - 80 = 0$$
$$x^2 - 2x - 20 = 0$$

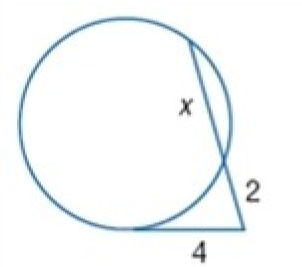
Use the Quadratic Formula to find the roots.

$$x = \frac{-(-2) \pm \sqrt{(2)^2 - 4(1)(-20)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{84}}{2}$$
$$\approx -3.6 \text{ or } 5.6$$

Since x is a length, it cannot be negative. Therefore, $x = 5.6$.

ANSWER:

5.6

52. 

SOLUTION:

If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

$$4^2 = 2(x + 2)$$
$$16 = 2x + 4$$
$$12 = 2x$$
$$6 = x$$

ANSWER:

6

53. **OPTIC ART** Victor Vasarely created art in the *op art* style. This piece, *AMBIGU-B*, consists of multi-colored parallelograms. Describe one method to ensure that the shapes are parallelograms.
Refer to the photo on Page 770.

SOLUTION:

Sample answer: If each pair of opposite sides are parallel, the quadrilateral is a parallelogram.

ANSWER:

Sample answer: If each pair of opposite sides are parallel, the quadrilateral is a parallelogram.

Evaluate each expression if *a* = 2, *b* = 6, and *c* = 3.

54. $\frac{1}{2}ac$

SOLUTION:

Substitute *a* = 2 and *c* = 3 in the expression.

$$\frac{1}{2}(2)(3) = 3$$

ANSWER:

3

55. $\frac{1}{2}cb$

SOLUTION:

Substitute $b = 6$ and $c = 3$ in the expression.

$\frac{1}{2}(3)(6) = 9$

ANSWER:

9

56. $\frac{1}{2}b(2a + c)$

SOLUTION:

Substitute $a = 2$, $b = 6$, and $c = 3$ in the expression.

$\frac{1}{2}(6)(2(2) + 3) = 21$

ANSWER:

21

57. $\frac{1}{2}c(b + a)$

SOLUTION:

Substitute $a = 2$, $b = 6$, and $c = 3$ in the expression.

$\frac{1}{2}(3)(6 + 2) = 12$

ANSWER:

12

58. $\frac{1}{2}a(2c + b)$

SOLUTION:

Substitute $a = 2$, $b = 6$, and $c = 3$ in the expression.

$\frac{1}{2}(2)(2(3) + 6) = 12$

ANSWER:

12