Experiment No: EM8

Experiment Name: Inductance of Solenoids

Objective: To determine the inductance of solenoids by using an LC circuit. To observe the length, cross sectional area and number of winding dependence of inductance of solenoids.

Keywords: Faraday's law of induction, Lenz's law, self and mutual inductance, LC oscillatory circuit,

Theoretical Information:

According to Britannica, inductance is defined as *a property of a conductor (often in the shape of a coil) that is measured by the size of the electromotive force, or voltage, induced in it, compared with the rate of change of the electric current that produces the voltage.*

If the inductance L does not depend on the current, this definition can be written in a mathematical language as,

$$V = L\frac{dI}{dt} \tag{8.1}$$

There are also some equivalent definitions of inductance. If we integrate above equation with assumption $L = const$ and remember Faraday's law of induction we would take,

$$\Phi = LI \tag{8.2}$$

where Φ is magnetic flux through the circuit. Thus, inductance can be seen as a measure of inductive property of a substance.

The SI unit of inductance is called the *henry* and is defined as the inductance of a conductor in which a current of 1 A flowing through it produces a flux linkage of 1 Wb.[1].

In the Gaussian system of units, the inductance has the dimension of length. Accordingly, the unit of inductance in this system is called *centimeter*. A loop with which a flux of 1 Mx (10^{-8} Wb) is linked to a current of 1 $cgsm_I$ (= 10 A) has an inductance of 1 cm.

Inductor is considered also as a passive device and is characterized by its reaction to change in current flowing through the circuit, obeying Faraday's law. This device, at last analysis, is a coil. Therefore, a solenoid can be considered as an inductor in a circuit. We will discuss this point at experimental part in detail.

Let's calculate the inductance of a solenoid, which is assumed to be long enough to ignore the end effects, with n turns per unit length, cross sectional area S and carrying a current I. As we know from Biot-Savart's law, magnetic induction inside this solenoid is constant and would be $B = \mu_0 \mu n I$. Here μ_0 and

[1] Wb: Weber, Mx: Maxwell

μ are magnetic permeabilities of vacuum and the medium, respectively. By definition, flux through each turn is BS and thus total magnetic flux linked with the solenoid is

$$\Phi = NBS = \mu_0 \mu n^2 Sl \cdot I \quad 8.3$$

from which the induction of the solenoid is found as,

$$L = \mu_0 \mu n^2 Sl. \quad 8.4$$

If we substitute $S = \pi r^2, n = N/l$ we would express L in terms of directly measurable quantities:

$$L = \mu_0 \mu \pi \frac{N^2 r^2}{l} \quad 8.5$$

This is the theoretical expression for induction of a cylindrical solenoid and it will become a reference point for your discussions.

<u>For your report, please submit from the beginning of the following page by showing calculations in reserved blank spaces and plotting graphics to the appropriate places.</u>

T.C.

GEBZE TECHNICAL UNIVERSITY

PHYSICS DEPARTMENT

PHYSICS LABORATORY II

EXPERIMENT REPORT

THE NAME OF THE EXPERIMENT :

DATE : ….. / ….. / ……….

PREPARED BY

NAME AND SURNAME:

STUDENT NUMBER :

GROUP NO :

DATE OF THE EXPERIMENT: ….. / ….. / ……….

Experimental Procedure:

There are a number of methods to measure the inductance of a substance. Here we will use the simplest and most basic one: an LC circuit. An LC circuit consists of a capacitor (C) and an inductor (L). Most basic property of this circuit is it's oscillatory nature. Charges inside it oscillate with a characteristic frequency:

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad 8.6$$

If the value of C is known, by measuring the frequency, the inductance, L, can be found from the expression below:

$$L = \frac{1}{4\pi^2 C f^2} \qquad 8.7$$

This is the experimental expression for inductance that you will need in experimental calculations.

As in mechanical oscillations, one should disturb the system in order to make it to oscillate. In an LC circuit we could do it in two ways: i) by initially charging the capacitor, ii) by initially allowing a magnetic flux to pass through the inductor.

We will prefer the second choice. After an oscillation starts, in principle (in the absence of resistance) it would continue permanently. But in practice, all electronic devices and circuit elements have a finite resistance. Therefore, the oscillations are damped, i.e. have diminishing amplitude with time. But no fear! We can anyway observe the oscillation by an analogue oscilloscope if we choose right circuit elements.

Instructions

1. Set the circuit as in the Figure 8.1.

2. Plug the capacitors of $C_1 = 1\ nF$ and $C_2 = 470\ pF$ in an parallel arrangement on apropriate place on the connection box. Note that black lines on the connection box shows the connected lines inside it. Internal capacitance of the oscilloscope is approximately 30 pF. Thus effective capacitance of the circuit is $C = 1\ nF + 470\ pF + 30\ pF = 1.5\ nF$.

3. Take two cables and wire the solenoid with unknown L and plug it in an apropriate place on the connection box to set the LC circuit given below.

4. Wire two cables from the two poles of capacitor and plug them into the first channel of the oscilloscope.

5. Turn the oscilloscope and function generator on, set the frequency of the latter to $f \cong 2000\ Hz$ and seek a good image on the former's display.

6. You should get an image like in Figure 8.2. This is the amplitude signal of the free damped oscillation in an LC circuit. The oscillation is due to resistance of all elements in the circuit, especially that of the oscilloscope. But this internal resistance is negligibly weak (there is only a %1 slip effect on the oscillation frequency) and we can assume that the oscillation frequency is equal to that of the ideal LC circuit:

$$\omega = \sqrt{\omega_0^2 - \beta^2} \approx \omega_0 \qquad 8.8$$

here ω_0 is the oscillation frequency of the ideal LC circuit, $\beta = R/2L \ll \omega_0$ is the damping coefficient and ω is the oscillation frequency of the RLC circuit.

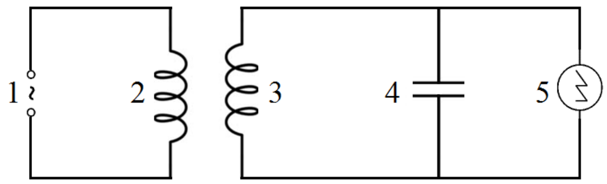


Figure 8.1

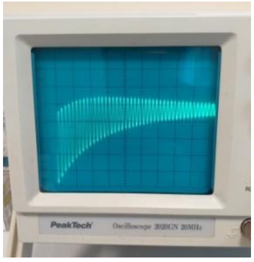


Figure 8.2

Figure 8.1: Schematic of the circuit.

1) AC current source,
2) Primary coil to trigger the oscillations in LC circuit,
3) Solenoid under investigation with unknown L,
4) Capacitor with $C = 1\,nF + 470\,pF + 30\,pF = 1.5\,nF$,
5) Oscilloscope to display and measure the voltage signal on the capacitor.

Figure 8.2: The picture showing the amplitude of free damped oscillations in the circuit.

7. Measure the horizontal distance between two peaks seen on the display. To do this you have to find the number of units between two successive peaks (N) and know the time factor (τ). The period of oscillatons is found as $T = N\tau$. During the experiment you will probably need to change the time factor, therefore you should be careful on its value in each measurement. You can read value of τ from the far right button on the front panel.

8. In the lab, only fill the first empty column in the table below. Other columns will be filled at home after calculations, first one by $f = 1/T$ and the second one after Equation 8.7.

Table 8.1:

Solenoid	N	$r\ (mm)$	$l\ (mm)$	$T\ (s)$	$f\ (s^{-1})$	$L\ (H)$
1	300	20	160			
2	300	16	160			
3	300	13	160			
4	200	20	105			
5	100	20	53			
6	150	13	160			
7	75	13	160			

$C = 470\,nF + 1\,pF + 30\,pF = 1.5\,nF$

Calculations

After the original table, you will need $log - log$ graphs to find winding, length and radius dependence of inductors. By using the previous table, create the following ones:

No	N	L	$\log N$	$\log L$
3				
6				
7				

No	l	L/N^s	$\log l$	$\log(L/N^s)$
1				
4				
5				

No	r	L	$\log r$	$\log L$
1				
2				
3				

Plot three data by using 4th and 5th columns for x and y axis respectively, and fit them with lines by least squares method and then find their slopes. As you will remember from the first semester lab courses, this analysis will give you powers of associated quantities ($m: s, q, p$) and thus, the dependence of the inductance on the chosen quantity[2]. Slope (m) is found from the following expression:

$$m = \frac{3\sum_{n=1}^{3} x_i y_i - \sum_{n=1}^{3} x_i \sum_{n=1}^{3} y_i}{\sum_{n=1}^{3} x_i^2 - (\sum_{n=1}^{3} x_i)^2} \quad 8.9$$

Here x_i and y_i are the logarithmic inputs in 4th and 5th columns taken from above three tables. For example, consider you found the slopes of lines generated from the tables as s, q, p respectively. That means the following expressions are valid:

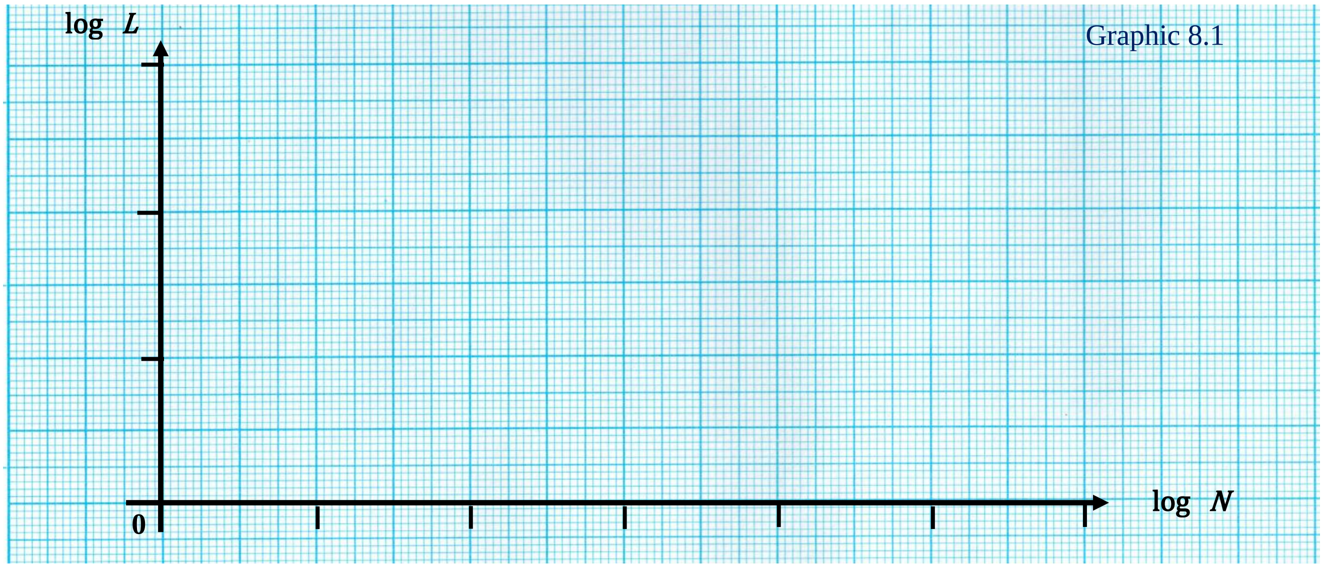
[2] Please refer to "Deneysel Metotlara Giriş" booklet written by Dr. Erbahar for the details of this analysis.

$$L \propto N^s; \ \frac{L}{N^s} \propto l^q \ ; \ L \propto r^p \tag{8.10}$$

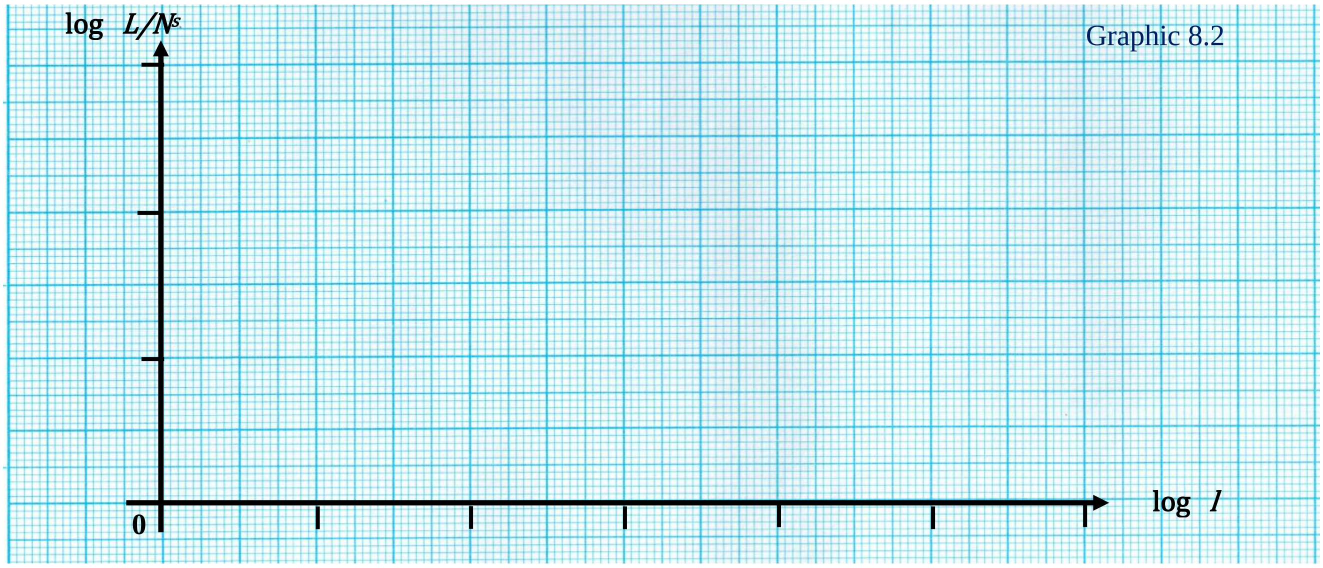
and as a result you will find

$$L = AN^s l^q r^p \tag{8.11}$$

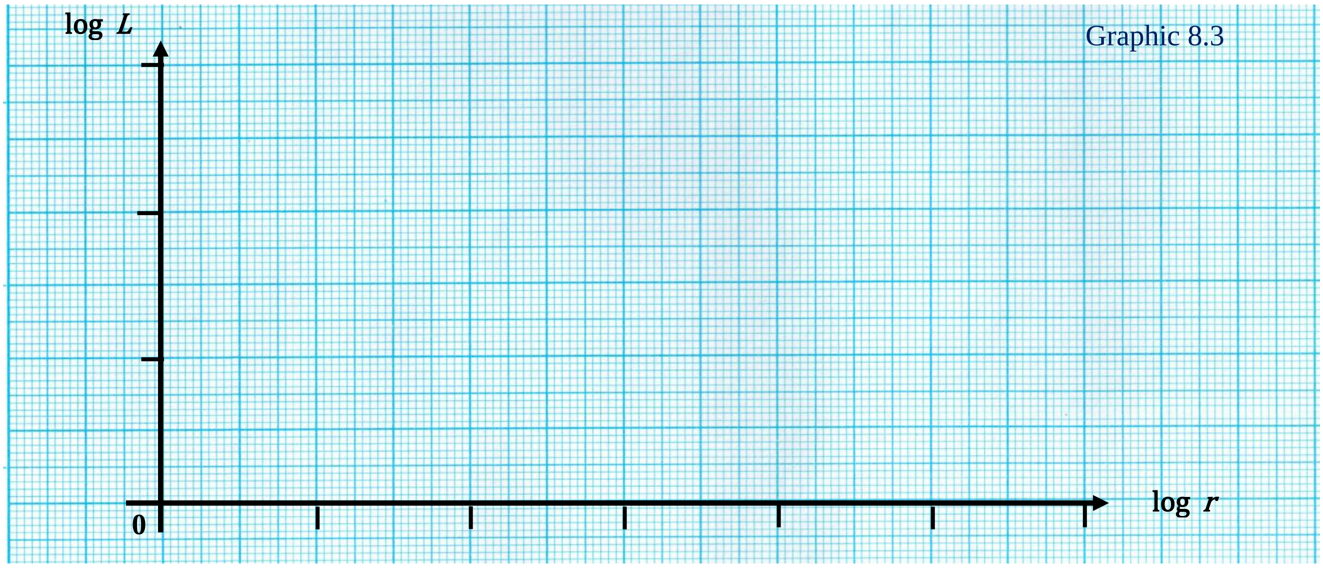
for inductance. Note that we have an idea about the values of the exponents as we have already derived the theoretical expression for it. In this way you will be able to create your own empirical formula for inductance.



Graphic 8.1



Graphic 8.2



Graphic 8.3

DISCUSSION

Compare the derived exponents by theoretical expression. Discuss and explain your results.

power	theory	fitting
s	2	
q	-1	
p	2	

Questions:

1. Derive the expression for frequency of a LC circuit.

2. Propose some other methods to find the inductance.

3. Find the derived units henry and farad in terms of basic quantities of SI system.

4. List the magnetic permeability and inductance values of some important materials: iron, steel, cobalt, aluminum, plastic, glass, water, air, vacuum etc. in two columns.

Sources

1. Young and Feedman, Sears and Zemansky's University Physics, 14th Edition, Pearson (2016).

2. For a simulation of RLC circuit and many of other systems:
http://www.vias.org/simulations/simusoft_dampedcircuit.html

3. A simulative explanation of LC oscillatory circuit from energy point of view and analogies with mechanics:
http://dev.physicslab.org/Document.aspx?doctype=3&filename=Induction_LCcircuits.xml

4. Erbahar D, "Deneysel Metotlara Giriş", GTÜ Fizik Bölümü (2015).
http://www.gtu.edu.tr/Files/UserFiles/90/Deneysel_metotlara_giri_1.pdf