Georgia
Standards of Excellence
Grade Level
Curriculum Overview

Mathematics

GSE Third Grade

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Georgia Standards of Excellence
Third Grade

****NEW** ***Click on the link in the table to view a video that shows instructional strategies for teaching the specified standard.***

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Numbers and Operations in Base Ten	**The Relationship Between Multiplication and Division**	**Patterns in Addition and Multiplication**	**Geometry**	**Representing and Comparing Fractions**	**Measurement**	**Show What We Know**
MGSE3.NBT.1 **MGSE3.NBT.2** **MGSE3.MD.3** **MGSE3.MD.4**	**MGSE3.OA.1** **MGSE3.OA.2** **MGSE3.OA.3** **MGSE3.OA.4** **MGSE3.OA.5** **MGSE3.OA.6** **MGSE3.OA.7** **MGSE3.NBT.3** **MGSE3.MD.3** **MGSE3.MD.4**	**MGSE3.OA.8** **MGSE3.OA.9** **MGSE3.MD.3** **MGSE3.MD.4** **MGSE3.MD.5** **MGSE3.MD.6** **MGSE3.MD.7**	**MGSE3.G.1** **MGSE3.G.2** **MGSE3.MD.3** **MGSE3.MD.4** **MGSE3.MD.7** **MGSE3.MD.8**	**MGSE3.NF.1** **MGSE3.NF.2** **MGSE3.NF.3** **MGSE3.MD.3** **MGSE3.MD.4**	**MGSE3.MD.1** **MGSE3.MD.2** **MGSE3.MD.3** **MGSE3.MD.4**	**ALL**
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units include the Mathematical Practices and indicate skills to maintain. However, the progression of the units is at the discretion of districts.						

Note: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 3-5 Key: G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, NF = Number and Operations, Fractions, OA = Operations and Algebraic Thinking.

STANDARDS FOR MATHEMATICAL PRACTICE

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.
In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.
Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

3. Construct viable arguments and critique the reasoning of others.
In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.
Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles

6. Attend to precision.
As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

7. Look for and make use of structure.
In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).

8. Look for and express regularity in repeated reasoning.
Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

<u>Mathematical Practices 1 and 6 should be evident in EVERY lesson***</u>***

CONTENT STANDARDS

OPERATIONS AND ALGEBRAIC THINKING (OA)

MGSE CLUSTER #1: REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. The terms students should learn to use with increasing precision with this cluster are: ***products, groups of, quotients, partitioned equally, multiplication****,* ***division, equal groups, arrays, equations****,* ***unknown.***

MGSE3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. ***For example, describe a context in which a total number of objects can be expressed as 5 × 7.***

The example given in the standard is one example of a *convention*, not meant to be enforced, nor to be assessed *literally*.

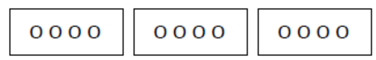
From the OA progressions document:
Page 25- "The top row of Table 3 shows the usual order of writing multiplications of Equal Groups in the United States. The equation 3x6 means how many are in 3 groups of 6 things each: three sixes. But in many other countries the equation 3 x6 means how many are 3 things taken 6 times (6 groups of 3 things each): six threes. Some students bring this interpretation of multiplication equations into the classroom. So, it is useful to discuss the different interpretations and allow students to use whichever is used in their home. This is a kind of linguistic commutativity that precedes the reasoning discussed above arising from rotating an array. These two sources of commutativity can be related when the rotation discussion occurs."
Also, the description of the convention in the standards is part of an "e.g.," to be used as an example of one way in which the standard might be applied. The standard itself says interpret the product. As long as the student can do this and explain their thinking, they've met the standard. It all comes down to classroom discussion and sense-making about the expression. Some students might say and see 5 taken 7 times, while another might say and see 5 groups of 7. Both uses are legitimate and the defense for one use over another is dependent upon a context and would be explored in classroom discussion.
Students won't be tested as to which expression of two equivalent expressions (2x5 or 5x2, for example) matches a visual representation. At most they'd be given 4 non-equivalent expressions to choose from to match a visual representation, so that this convention concern wouldn't enter the picture.
Bill McCallum has his say about this issue, here: http://commoncoretools.me/forums/topic/3-oa-a/"

MGSE3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares (How many in each group?), or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each (How many groups can you make?).

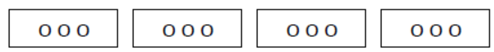
For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.

Partition models focus on the question, “How many in each group?” A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?



Measurement (repeated subtraction) models focus on the question, “How many groups can you make?” A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?



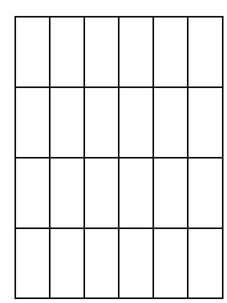
MGSE3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary: *Multiplication and Division Within 100*.

This standard references various strategies that can be used to solve word problems involving multiplication and division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many brownies does each person receive? *(4 × 9 = 36, 36 ÷ 6 = 6).*

Table 2, located at the end of this document, gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.

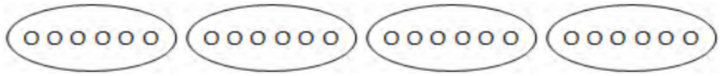
Examples of multiplication: There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?

This task can be solved by drawing an array by putting 6 desks in each row.
This is an array model:



This task can also be solved by drawing pictures of equal groups.

4 groups of 6 equals 24 objects

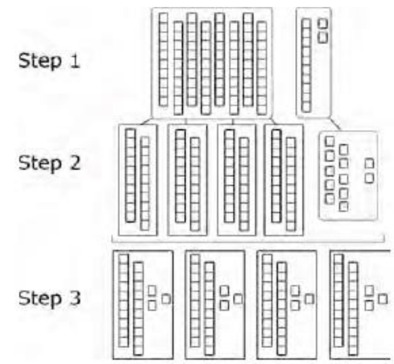


A student could also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom." A number line could also be used to show equal jumps. Third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables).. Letters are also introduced to represent unknowns in third grade.

Examples of division: There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class. *(□ ÷ 4 = 6. There are 24 students in the class).*

Determining the number of objects in each share (partitive division, where the size of the groups is unknown):

Example: The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



Determining the number of shares (measurement division, where the number of groups is unknown):

Example: Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**
24	24 – 4 = 20	20 – 4 = 16	16 – 4 = 12	12 – 4 = 8	8 – 4 = 4	4 – 4 = 0

Solution: The bananas will last for 6 days.

MGSE. 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers using the inverse relationship of multiplication and division.

For example, determine the unknown number that makes the equation true in each of the equations, 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.

This standard refers Table 2, included at the end of this document for your convenience, and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product ($3 \times 6 = ?$ or $18 \div 3 = 6$). The more difficult problem structures include Group Size Unknown ($3 \times ? = 18$ or $18 \div 3 = 6$) or Number of Groups Unknown ($? \times 6 = 18$, $18 \div 6 = 3$). The focus of MGSE3.OA.4 goes beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division.

Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given $4 \times ? = 40$, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Example: Solve the equations below:

- $24 = ? \times 6$
- $72 \div \Delta = 9$
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether?

  $3 \times 4 = m$

<u>MGSE CLUSTER #2:</u> UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, strategies, (properties)-rules about how numbers work.

MGSE3.OA.5 Apply properties of operations as strategies to multiply and divide.

Examples:

If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)

3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.)

Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

This standard references properties (rules about how numbers work) of multiplication. While students DO NOT need to use the formal terms of these properties, students should understand that properties are rules about how numbers work, they need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 × 5 × 2, a student could rearrange the numbers to first multiply 5 × 2 = 10 and then multiply 10 × 7 = 70.

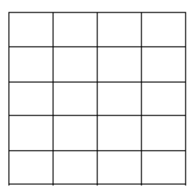
The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that 5 × 4 = 20, then they also know that 4 × 5 = 20. The array below could be described as a 5 × 4 array for 5 columns and 4 rows, or a 4 × 5 array for 4 rows and 5 columns. There is no "fixed" way to write the dimensions of an array as rows × columns or columns × rows.
Students should have flexibility in being able to describe both dimensions of an array.

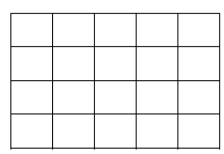
Example:

4 × 5

or

5 × 4





4 × 5

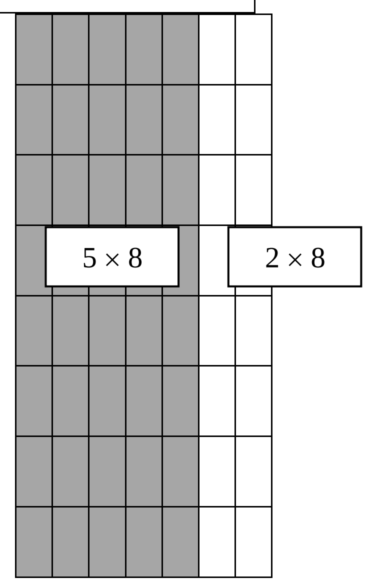
or

5 × 4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7 × 6. Again, students should use the distributive property, but can refer to this in informal language such as "breaking numbers apart".

Student 1	**Student 2**	**Student 3**
7×6	7×6	7×6
$7 \times 5 = 35$	$7 \times 3 = 21$	$5 \times 6 = 30$
$7 \times 1 = 7$	$7 \times 3 = 21$	$2 \times 6 = 12$
$35 + 7 = 42$	$21 + 21 = 42$	$30 + 12 = 42$

Another example if the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem $7 \times 8 = ?$, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying $5 \times 8 = 40$ and $2 \times 8 = 16$ and adding the two products ($40 + 16 = 56$).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- $0 \times 7 = 7 \times 0 = 0$ (Zero Property of Multiplication)
- $1 \times 9 = 9 \times 1 = 9$ (Multiplicative Identity Property of 1)
- $3 \times 6 = 6 \times 3$ (Commutative Property)
- $8 \div 2 = 2 \div 8$ (Students are only to determine that these are not equal)
- $2 \times 3 \times 5 = 6 \times 5$
- $10 \times 2 < 5 \times 2 \times 2$
- $2 \times 3 \times 5 = 10 \times 3$
- $0 \times 6 > 3 \times 0 \times 2$

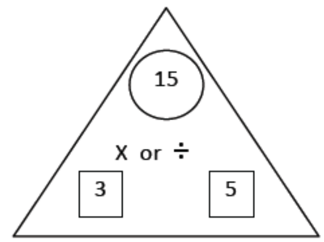
MGSE3.OA.6 Understand division as an unknown-factor problem.

For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

This standard refers to Table 2, included at the end of this document for your convenience, and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.



Examples:

- 3 × 5 = 15 5 × 3 = 15
- 15 ÷ 3 = 5 15 ÷ 5 = 3

<u>MGSE CLUSTER # 3</u>: MULTIPLY AND DIVIDE WITHIN 100.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property.

MGSE3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

This standard mentions the word *fluently* when students are multiplying and dividing numbers within 100. Fluency means accuracy (attending to precision), efficiency (using well-understood strategies with ease), and flexibility (using strategies such as the distributive property). Research indicates that teachers can best support students' development of automaticity with sums and differences through varied experiences making 10, breaking numbers apart and working on mental strategies, rather than timed tests. "Know from memory" should not focus on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 × 9).

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts.

Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

According to NCTM, fluency is also the ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop fluency, students need experience in integrating concepts and strategies and building on familiar strategies as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through strategic practice. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014).

Strategies students may use to attain fluency include:

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5 × 10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of __ and knowing how many groups have been counted)
- Square numbers (ex: 3 × 3)
- Nines (10 groups less one group, e.g., 9 × 3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6 × 7 is 6 x 6 plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex: 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24)
- Missing factors

General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

Research indicates that teachers' can best support students' development of automaticity with computation through varied experiences with part-whole reasoning, breaking numbers apart and working on mental strategies, rather than timed tests. Evidence from research has indicated that timed tests cause unhealthy math anxiety with learners as they are developing a solid foundation in numeracy: https://www.youcubed.org/resources/new-evidence-timed-test-teaching-children-mathematics-april-2014/.

<u>MGSE CLUSTER #4</u>: SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties) – rules about how numbers work.

MGSE3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

See Glossary, Table 2

**IMPORTANT INFORMATION ON ORDER OF OPERATIONS FOUND HERE: Order of Operations: The Myth and The Math (Please read BEFORE beginning this unit.)

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities. Footnote 3 within the 3rd grade standards document indicates that students should know how to perform operations in the conventional order where there are no parentheses to specify a particular order. Therefore, they should learn the convention for order of operations. **Avoid the rote memorization of PEMDAS**, as this mnemonic can cause confusion for students who memorize without understanding and thus assume multiplication *must* occur before division and/or addition *must* occur before subtraction. Not so- they are performed in the order in which they occur in an expression, with multiplication/division occurring prior to addition/subtraction.

For example, many learners may believe that the expression 16÷4(4) is equal to 1. This misconception is fostered through the use of a procedure like PEMDAS without understanding. Further clarification from the OA Progressions document: Students in Grade 3 begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two-step problems (3.OA.8). But the symbols of arithmetic, x or • for multiplication and ÷ or / for division, continue to be used in Grades 3, 4, and 5. Understanding and using the associative and distributive properties (as discussed above) requires students to know two conventions for reading an expression that has more than one operation: 1. Do the operation inside the parentheses before an operation outside the parentheses (the parentheses can be thought of as hands curved around the symbols and grouping them). 2. If multiplication or division is written first, next to addition or subtraction, imagine parentheses around the multiplication or division (it is done before these operations). At Grades 3 through 5, parentheses can usually be used for such cases so that fluency with this rule can wait until Grade 6. These conventions are often called the Order of Operations and can seem to be a central aspect of algebra. However, they actually are just simple "rules of the road" that allow expressions involving more than one operation to be interpreted unambiguously and thus are connected with the mathematical practice of communicating precisely, SMP6. Use of parentheses is important in displaying structure and thus is connected with the mathematical practice of making use of structure, SMP7. Parentheses are important in expressing the associative and especially the distributive properties. These properties are at the heart of Grades 3 to 5 because they are used in multiplication and division strategies, in multi-digit and decimal multiplication and division, and in all operations with fractions.

Example:

> Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution. One possible solution: $2 \times 5 + m = 25$.

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example:

> On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?
>
> Here are some typical estimation strategies for the problem:

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then, I have 67 in 267 and the 34. When I put 67 and 34 together, that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

MGSE3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.‡ ***For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.***

See Glossary, Table 3

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations.

Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).
- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What do you notice about the numbers highlighted in pink in the multiplication table? Explain a pattern using properties of operations. *When one changes the order of the factors (commutative property), one still gets the same product; example 6 x 5 = 30 and 5 x 6 = 30.*

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.

Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
□	□	□
□	□	□
□	□	□
20	0	20

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

NUMBERS AND OPERATIONS IN BASE TEN (NBT)

MGSE CLUSTER: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties)-rules about how numbers work.

MGSE3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

MGSE3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

This standard mentions the word *fluently* when students are adding and subtracting numbers within 1000. Fluency means accuracy (attending to precision), efficiency (using well-understood strategies with ease), and flexibility (using strategies such as the distributive property). According to NCTM, fluency is also the ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop fluency, students need experience in integrating concepts and strategies and building on familiar strategies as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through strategic practice. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014).

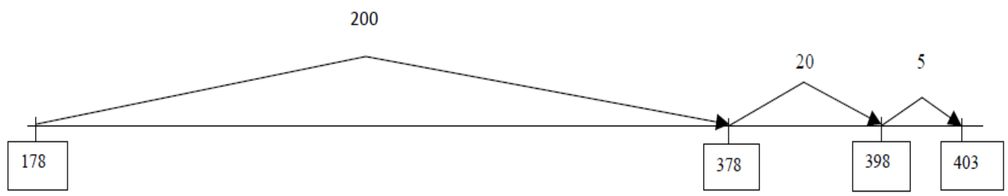
The word algorithm refers to a procedure or a series of steps. In third grade, students should be able to flexibly use a variety of algorithms based on their understanding. The process of students standardizing and becoming fluent with their algorithm is reserved for 4th grade with standard 4.NBT.4. Third grade students should have experiences with a variety of strategies and algorithms. Mathematical concepts will be assessed in a way where a variety of algorithms can be used based on what the student understands.

Problems should include both horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.

Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1	Student 2	Student 3
100 + 200 = 300 70 + 20 = 90 8 + 5 = 13 300+90+13 = 403 students	I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.	I know the 75 plus 25 equals 100. Then, I added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So, I have 4 hundreds plus 3 more which is 403.

Student 4
178 + 225 = ?
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403



MGSE3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

This standard extends students' work in multiplication by having them apply their understanding of place value.

This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200.

<u>Common Misconceptions</u>

The use of terms like "round up" and "round down" confuses many students. For example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

NUMBER AND OPERATIONS IN FRACTIONS (NF)

MGSE CLUSTER: DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS.

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

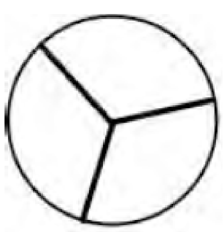
MGSE3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts (unit fraction); understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. *For example,* $\frac{3}{4}$ *means there are three* $\frac{1}{4}$ *parts, so* $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

This standard refers to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares), set models (parts of a set), and number lines. To show mastery of 3.NF.1, students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1. For example, the fraction 3/5 is composed of 3 pieces that each have a size of 1/5.

Some important concepts related to developing understanding of fractions include:

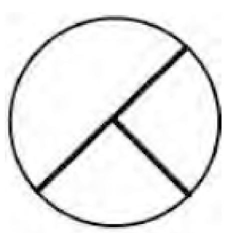
- Understand fractional parts must be equal-sized.

Example



These are thirds.

Non-Example



These are NOT thirds.

- The number of equal parts tells how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.

- The number of children in one-half of a classroom is different than the number of children in one-half of a school. (The whole in each set is different; therefore, the half in each set will be different.)
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
    - ¾ means that there are 3 one-fourths.
    - Students can count *one fourth, two fourths, three fourths.*
      Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop a conceptual understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.

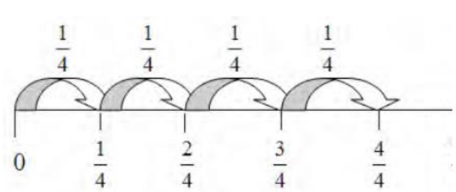
MGSE3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. **Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$. Recognize that a unit fraction $\frac{1}{b}$ is located $\frac{1}{b}$ whole unit from 0 on the number line.**

b. **Represent a non-unit fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths of $\frac{1}{b}$ (unit fractions) from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the non − unit fraction $\frac{a}{b}$ on the number line.**

The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that ½ is between 0 and 1).

In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼. (**MGSE3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction 3/4. (**MGSE3.NF.2b**).



MGSE3.NF.3 Explain equivalence of fractions through reasoning with visual fraction models. Compare fractions by reasoning about their size.

An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than 1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions with denominators of 2, 3, 4, 6, and 8, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$**. Explain why the fractions are equivalent, e.g., by using a visual fraction model.**

These standards call for students to use visual fraction models (e.g, area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. ***Examples: Express 3 in the form 3 =*** $\frac{6}{2}$ $\boldsymbol{(3\ wholes\ is\ equal\ to\ six\ halves)}$; $\boldsymbol{recognize\ that}$ $\frac{3}{1} = \mathbf{3}$; $\boldsymbol{locate}$ $\frac{4}{4}$ ***and 1 at the same point of a number line diagram.***

This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1.

Example: If 6 brownies are shared between 2 people, how many brownies would each person get?

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than 1/4 of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, 1/2 of a large pizza is a different amount than 1/2 of a small pizza. Students should be given opportunities to discuss and reason about which 1/2 is larger.

Common Misconceptions

The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is. The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.

Students think all shapes can be divided the same way. Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:

1. Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
2. Fold in this manner two more times.
3. Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

The idea of referring to a collection as a single entity makes set models difficult for some children. Students will frequently focus on the size of the set rather than the number of equal sets in the whole. For example, if 12 counters make a whole, then a set of 4 counters is one-*third*, not one-fourth, since 3 equal sets make the whole. However, the set model helps establish important connections with many real-world uses of fractions, and with ratio concepts.

Models for Fractions

(excerpted/adapted from "Elementary and Middle School Mathematics: Teaching Developmentally" Van de Walle, Karp, Bay-Williams; 7th edition, pp 228-291)

The use of models in fraction tasks allows students to clarify ideas which are often confused in a purely symbolic mode. Different models allow different opportunities to learn. For example, an area model helps students visualize parts of the whole. A linear model shows that there is always another fraction to be found between any two fractions- an important concept that is underemphasized in the teaching of problems. Also, some students are able to make sense of one model, but not another. Using appropriate models and using models of each type broaden and deepen students (and teachers) understanding of fractions. It is important to remember that students must be able to explore fractions across models. If they never see fractions represented as a length, they will struggle to solve any problem or context that is linear. As a teacher, you will not know if they really understand the meaning of a fraction such as ¼ unless you have seen a student model one-fourth using different contexts and models.

3 Categories of Models

Region or Area Models- In all of the sharing tasks, the tasks involve sharing something that could be divided into smaller parts. The fractions are based on parts of an area or region. There are many region models, including: circular pieces, rectangular regions, geoboards, grids or dot paper, pattern blocks, tangrams, paper folding. Students should have been exposed to "sharing" by partitioning shapes into halves and fourths in 1st and 2nd grades.

Length Models- With length models, lengths or measurements are compared instead of areas. Either lines are drawn and subdivided (number lines, measuring tapes, rulers), or physical materials are compared on the basis of lengths (Cuisenaire rods or fraction strips).
Rods or strips provide flexibility because any length can represent the whole. The number line is a more sophisticated model, and an essential one that should be emphasized more in the teaching of fractions. Linear models are more closely connected to the real-world contexts in which fractions are commonly used- measuring. Importantly, the number line reinforces that there is always one more fraction to be found between two fractions.

Set Models- In set models, the whole is understood to be a set of objects, and subsets of the whole make up fractional parts. For example, 3 objects are one-fourth of a set of 12 objects. The set of 12, in this example, represents the whole, or 1. Set models can be created using actual objects (such as toy cars and trucks) or two-color counters.

MEASUREMENT AND DATA (MD)

MGSE CLUSTER #1: SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION OF INTERVALS OF TIME, LIQUID VOLUMES, AND MASSES OF OBJECTS.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L).

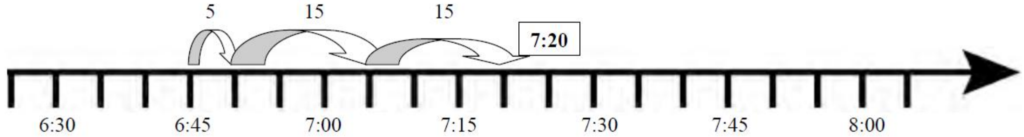
MGSE. 3.MD.1 Tell and write time to the nearest minute and measure elapsed time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram, drawing a pictorial representation on a clock face, etc.

This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).

Example:

Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed,

and 15 minutes to eat breakfast. What time will she be ready for school?



MGSE3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2]
This standard asks for students to reason about the units of mass and volume. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Word problems should only be one-step and include the same units.

Example:

> Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram.

Example:

> A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams?

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principle).

Common Misconceptions

Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams may be read as 81 grams. Students realize it is one away from 80, but do not think of it as 79 grams.

[1] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"). See Table 2 at the end of this document.

<u>MGSE CLUSTER #2</u>: REPRESENT AND INTERPRET DATA.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: scale, scaled picture graph, scaled bar graph, line plot, data.

MGSE3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

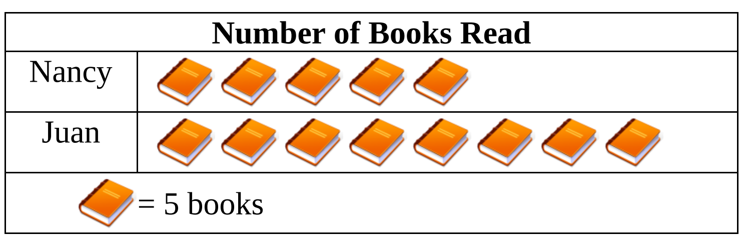
Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should **P**ose a question, **C**ollect data, **A**nalyze data, and **I**nterpret data (PCAI). Students should be graphing data that is relevant to their lives

Example:

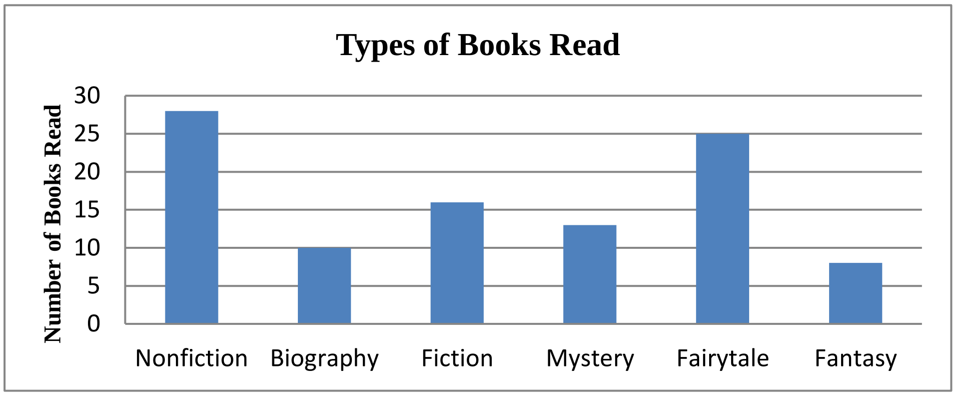
Pose a question: Student should come up with a question. What is the typical genre read in our class?

Collect and organize data: student survey

<u>Pictographs</u>: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



<u>Single Bar Graphs</u>: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.

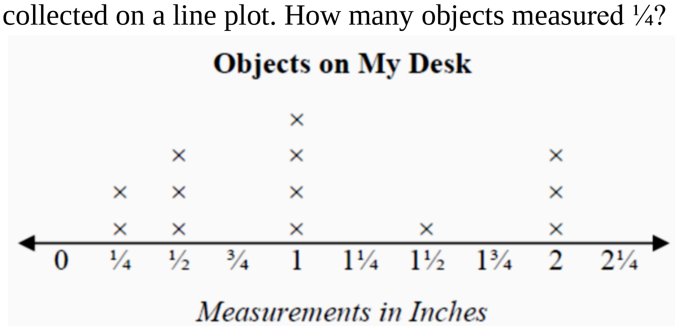


Analyze and Interpret data:

- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre? Why?

MGSE3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Students in second grade measured length in whole units using both metric and U.S. customary systems. It is important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment. This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch. Example: Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc. …



Common Misconceptions

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip-counting when determining the value of a bar since the scale is not in single units.

MGSE CLUSTER #3: GEOMETRIC MEASUREMENT – UNDERSTAND CONCEPTS OF AREA AND RELATE AREA TO MULTIPLICATION AND TO ADDITION.

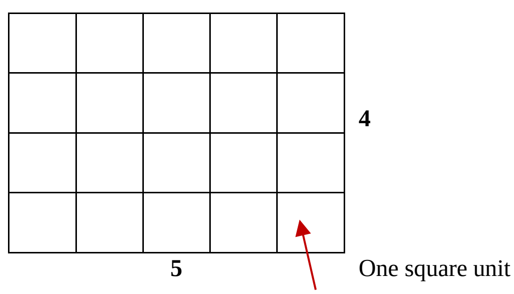
Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attribute, area, square unit, plane figure, gap, overlap, square cm, square m, square in., square ft, nonstandard units, tiling, side length, decomposing.

MGSE3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper.



MGSE3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.

MGSE3.MD.7 Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles then multiply their side lengths to show it is the same.

To find the area, one could count the squares or multiply 3 × 4 = 12.

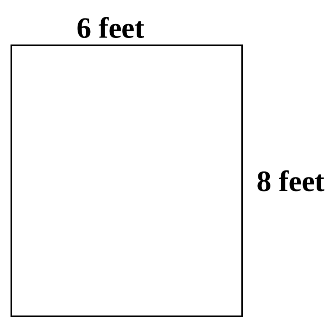
1	2	3	4
5	6	6	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

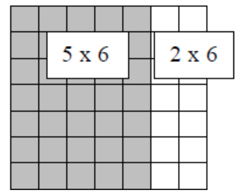
Example:

Drew wants to tile the bathroom floor using 1-foot tiles. How many square foot tiles will he need?

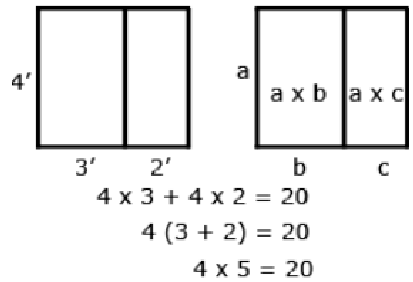


c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 × 6 figure can be determined by finding the area of a 5 × 6 and 2 × 6 and adding the two sums.



Example:



Common Misconceptions

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

<u>MGSE CLUSTER #4</u>: GEOMETRIC MEASUREMENT – RECOGNIZE PERIMETER AS AN ATTRIBUTE OF PLANE FIGURES AND DISTINGUISH BETWEEN LINEAR AND AREA MEASURES.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***attribute, perimeter, plane figure, linear, area, polygon, side length.***

MGSE3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Common Misconceptions

Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter. Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.

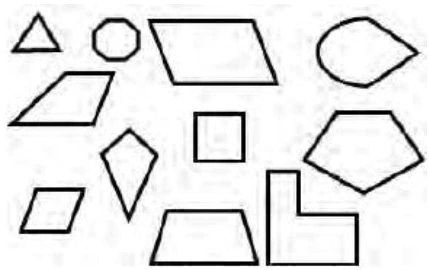
GEOMETRY (G)

MGSE CLUSTER: REASON WITH SHAPES AND THEIR ATTRIBUTES.

Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attributes, properties, quadrilateral, open figure, closed figure, three-sided, 2-dimensional, 3-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, cubes, cones, cylinders, and rectangular prisms are subcategories of 3-dimensional figures shapes: polygon, rhombus/rhombi, rectangle, square, partition, unit fraction. From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere.

MGSE3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.



Students should classify shapes by attributes and drawing shapes that fit specific categories. For example, parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.

Example:

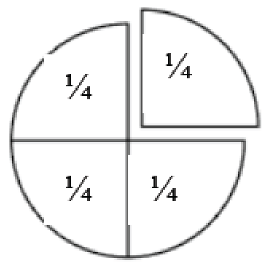
> Draw a picture of a quadrilateral. Draw a picture of a rhombus. How are they alike? How are they different? Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

MGSE3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. ***For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.***

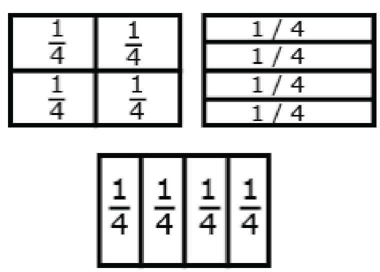
This standard builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths.

Example:

> This figure was partitioned/divided into four equal parts. Each part is ¼ of the total area of the figure.



Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.



Common Misconceptions

Students may identify a square as a "nonrectangle" or a "nonrhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Table 1

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put together/ Take apart[4]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[5]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[5] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2

Common Multiplication and Division Situations

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

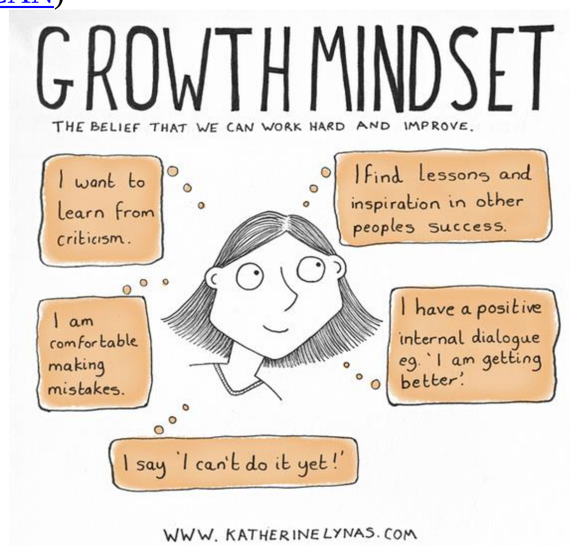
	Unknown Product	**Group Size Unknown** ("How many in each group? Division)	**Number of Groups Unknown** ("How many groups?" Division)
	3 × 6 = ?	**3 × ? – 18, and 18 ÷3 = ?**	**? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[6], **Area[7]**	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[6] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[7] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

MINDSET AND MATHEMATICS

Growth mindset was pioneered by Carol Dweck, Lewis and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability.
(from WITH+MATH=I CAN)



Jo Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education and author of *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*, was one of the first to apply growth mindset to math achievement.
You can learn how to use the power of growth mindset for yourself and your students here:

https://www.amazon.com/gp/withmathican

https://www.mindsetkit.org/topics/about-growth-mindset

https://www.youcubed.org/

Growth and Fixed Mindset images courtesy of Katherine Lynas (katherinelynas.com). Thank you, Katherine!

VERTICAL UNDERSTANDING OF THE MATHEMATICS LEARNING TRAJECTORY

Why does it matter if you know what happens in mathematics in the grades before and after the one you teach? Isn't it enough just to know and understand the expectations for your grade?

There are many reasons to devote a bit of your time to the progression of standards.

You will:

- Deepen your understanding of how development of algebraic thinking has proven to be a critical element of student mathematics success as they transition from elementary to middle school. Elementary and middle school teachers must understand how algebraic thinking develops prior to their grade, in their grade, and beyond their grade in order to support student algebraic thinking
- Know what to expect when students show up in your grade because you know what they should understand from the years before
- Understand how conceptual understanding develops, making it easier to help students who have missing bits and pieces
- Be able to help students to see the connections between ideas in mathematics in your grade and beyond, helping them to connect to what they already know and what is to come
- Assess understanding more completely, and develop better assessments
- Know what the teachers in the grades to come expect your students to know and understand
- Plan more effectively with same-grade and other-grade colleagues
- Deepen your understanding of the mathematics of your grade

We aren't asking you to take a month off to study up, just asking that you reference the following resources when you want to deepen your understanding of where students are in their mathematics learning, understand why they are learning what they are learning in your grade, and understand the mathematical ideas and connections within your grade and beyond.

Resources:

The Coherence Map:

http://achievethecore.org/page/1118/coherence-map This resource diagrams the connections between standards, provides explanations of standards, provides example tasks for many standards, and links to the progressions document when further detail is required.

A visual learning trajectory of:

Addition and Subtraction - http://gfletchy.com/2016/03/04/the-progression-of-addition-and-subtraction/
Multiplication - http://gfletchy.com/2015/12/18/the-progression-of-multiplication/
Division - http://gfletchy.com/2016/01/31/the-progression-of-division/
Fractions - https://gfletchy.com/2016/12/08/the-progression-of-fractions/
(Many thanks to Graham Fletcher, the genius behind these videos)

Learning Trajectories:

https://www.mobt3ath.com/uplode/book/book-24726.pdf

https://repository.upenn.edu/cgi/viewcontent.cgi?article=1019&context=cpre_researchreports

The Mathematics Progression Documents:

http://math.arizona.edu/~ime/progressions/

RESEARCH OF INTEREST TO MATHEMATICS TEACHERS:

Social Emotional Learning and Math-
http://curry.virginia.edu/uploads/resourceLibrary/Teachers_support_for_SEL_contributes_to_improved_math_teaching_and_learning,_et_al.pdf

Why how you teach math is important- https://www.youcubed.org/

GloSS, IKAN and the overall Numeracy Project

Information on the Numeracy Project, which includes GloSS and IKAN can be found here: Georgia Numeracy Project Overview.

The GloSS and IKAN professional learning video found here: https://www.georgiastandards.org/Georgia-Standards/Pages/FOA/Foundations-of-Algebra-Day-1.aspx provides an in-depth look at the GloSS and IKAN. While it was created for teachers of Foundations of Algebra, the information is important for teachers of grades K- 12.

The GloSS and IKAN prezi found on georgiastandards.org, here: https://www.georgiastandards.org/Georgia-Standards/Pages/Global-Strategy-Stage-GloSS-and-Individual-Knowledge-Assessment-of-Number-IKAN.aspx

FLUENCY:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit. Fluency, however, is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: Memorization leads to the rapid recall of arithmetic facts or mathematical procedures without the necessity of understanding. This type of learning is not the goal of numeracy. Memorization is often confused with fluency and automaticity. Fluency implies a much richer kind of mathematical knowledge and experience. Automaticity is based on strategy development and the ability to become automatic with part-whole computation strategies developed.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and automaticity.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing-there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task.
Clarify expectations for products/behavior.
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task
- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Let them Grapple!
Students- grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe and assess.

Closing: Best Learning Happens Here
Students- share answers, justify thinking, clarify understanding, explain thinking, question each other
Teacher- Listen attentively to all ideas, ask for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Anchor charts developed to record big ideas/strategies to be carried forward.
Read Van de Walle K-3, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

1. Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the

tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html (this is a for-profit site with several free resources)
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, that does not mean that these are the only standards addressed in the task. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school; most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem-solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. It is important for you to understand the difference between story problems and context problems. "Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around many problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult; however; there are excellent reasons for making the effort.

- Problem solving focuses students' attention on ideas and sense making
- Problem solving develops the belief in students that they are capable of doing the mathematics and that mathematics makes sense
- Problem solving provides on going assessment data
- Problem solving is an excellent method for attending to a breadth of abilities
- Problem solving engages students so that there are few discipline problems
- Problem solving develops "mathematical power"
  (Van de Walle 3-5 pg. 15 and 16)

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are, which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 11 and 12

Use of Manipulatives

Used correctly manipulatives can be a positive factor in children's learning. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 6

When a new model of new use of a familiar model is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use.

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty. (Van de Walle and Lovin, Teaching Student-Centered Mathematics3-5 pg. 9
- Modeling also includes the use of mathematical symbols to represent/model the concrete mathematical idea/thought process/situation. This is a very important, yet often neglected step along the way. Modeling can be concrete, representational, and abstract. Each type of model is important to student understanding. Modeling also means to "mathematize" a situation or problem, to take a situation which might at first glance not seem mathematical, and view it through the lens of mathematics. For example, students notice that the cafeteria is always out of their favorite flavor of ice cream on ice cream days. They decide to survey their schoolmates to determine which flavors are most popular, and share their data with the cafeteria manager so that ice cream orders reflect their findings. The problem: Running out of ice cream flavors. The solution: Use math to change the flavor amounts ordered.

<u>Use of Strategies and Effective Questioning</u>

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking 'Guess what is in my head' questions;
- focusing on just a small number of learners;
- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A 'no hands' approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient 'wait time' between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

<u>Number Lines</u>

The use of number lines in elementary mathematics is crucial in students' development of number and mathematical proficiency. While the GSE explicitly state use number lines in grades 2-5, number lines should be used in all grade levels and in multiple settings.

According to John Van de Walle,

> A number line is also a worthwhile model, but can initially present conceptual difficulties for children below second grade and students with disabilities (National Research Council Committee, 2009) This is partially due to their difficulty in seeing the unit, which is a challenge when it appears in a continuous line. A number line is also a shift from counting a number of individual objects in a collection to continuous length units. There are, however, ways to introduce and model number lines that support young learners as they learn this representation. Familiarity with a number line is essential because third grade students will use number lines to locate fractions and add and subtract time intervals, fourth graders will locate decimals and use them for measurement, and fifth graders will use perpendicular number lines in coordinate grids (CCSSO, 2010).
>
> A number line measures distance from zero the same way a ruler does. If you don't actually teach the use of the number line through emphasis on the unit (length), students may focus on the hash marks or numerals instead of the spaces (a misunderstanding that becomes apparent when their answers are consistently off by one). At first students can build a number path by using a given length, such as a set of Cuisenaire rods of the same color to make a straight line of multiple single units (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 106-107)

Open number lines are particularly useful for building students' number sense. They can also form the basis for discussions that require the precise use of vocabulary and quantities, and are therefore a good way to engage students in the Standards for Mathematical Practice.

While the possibilities for integrating number lines into the mathematics classroom are endless, the following are some suggestions/ideas:

- On a bulletin board, attach a string which will function as an open number line. Each morning (or dedicated time for math routines) put a new number on each student's desk.

Using some type of adhesive (thumb tack, tape, etc.), students will place the number in the appropriate location on the string. In the beginning of the year, provide students with numbers that are more familiar to them. As the year progresses, move through more complex problems such as skip counting, fractions, decimals or other appropriate grade level problems. Through daily integration, the number line becomes part of the routine. Following the number placement, have a brief discussion/debriefing of the reasoning used by students to place the numbers.

- In the 3-Act tasks placed throughout the units, students will be provided opportunities to use an open number line to place estimates that are too low, too high and just right as related to the posed problem. Similar opportunities can also be used as part of a daily routine.

Math Maintenance Activities

In addition to instruction centered on the current unit of study, the math instructional block should include time devoted to reviewing mathematics that have already been taught, previewing upcoming mathematics, and developing mental math and estimation skills. There is a saying that if you don't use it, you'll lose it. If students don't have opportunities to continuously apply and refine the math skills they've learned previously, then they may forget how to apply what they've learned. Unlike vocabulary words for literacy, math vocabulary words are not used much outside math class, so it becomes more important to use those words in discussions regularly. Math maintenance activities incorporate review and preview of math concepts and vocabulary and help students make connections across domains. It's recommended that 15 to 30 minutes of the math instructional block be used for these math maintenance activities each day. It's not necessary nor is it recommended that teachers do every activity every day. Teachers should strive for a balance of math maintenance activities so that over the course of a week, students are exposed to a variety of these activities. Math maintenance time may occur before or after instruction related to the current math unit, or it can occur at a different time during the day.

The goals of this maintenance time should include:

- Deepening number sense, including subitizing, flexible grouping of quantities, counting forward and backward using whole numbers, fractions, decimals and skip counting starting at random numbers or fractional amounts
- Developing mental math skills by practicing flexible and efficient numerical thinking through the use of operations and the properties of operations
- Practicing estimation skills with quantities and measurements such as length, mass, and liquid volume, depending on grade level
- Practicing previously-taught skills so that students deepen and refine their understanding
- Reviewing previously-taught concepts that students struggled with as indicated on their assessments, including gaps in math concepts taught in previous grade levels
- Using a variety of math vocabulary terms, especially those that are used infrequently

- Practicing basic facts using strategies learned in previous grade levels or in previous units to develop or maintain fluency
- Previewing prerequisite skills for upcoming math units of study
- Participating in mathematical discussions with others that require students to construct viable arguments and critique the reasoning of others

To accomplish these goals, math maintenance activities can take many different forms. Some activities include:

- Number Corner or Calendar Time
- Number Talks
- Estimation Activities/Estimation 180
- Problem of the Day or Spiraled Review Problems

In addition, math discussions, math journals and math games are appropriate not only for the current unit of study, but also for maintaining math skills that were previously taught.

Although there are commercially-available materials to use for math maintenance activities, there are also many excellent websites and internet resources that are free for classroom use. Here is a partial list of some recommended resources. A more detailed explanation of some of these components follows below.

Math Maintenance Activity	**Possible Resources**
Number Corner or Calendar Time	• http://www.teachinginroom6.com/p/calendar-math.html - blog post from a teacher who uses calendar (with ideas and resources) • Number Corner from The Math Learning Center
Number Talks	• *Number Talks* by Sherry Parrish • http://kentuckymathematics.org/number_talk_resources.php
Estimation Activities/Estimation 180	• http://www.estimation180.com/
Problem of the Day/Spiraled Review Problems	• www.insidemathematics.org • http://nzmaths.co.nz/teaching-material • http://www.k-5mathteachingresources.com/ • *Children's Mathematics: Cognitively Guided Instruction* by Thomas Carpentar, et. Al. • *Extending Children's Mathematics: Fractions and Decimals* by Epson and Levi

Number Corner/Calendar Time

Number Corner/Calendar Time is an interactive routine centered on a monthly calendar in which students can use the date or day of the school year as a starting point for math discussions. Typically, there are date cards for each day of the month with repeating or growing patterns. The patterns can be numeric or geometric and can lead to rich discussions of math vocabulary and predictions of what the next date card will look like. The day of the school year can be used as a basis for discussions of patterns on the 100's chart, or the days can each be multiplied by 0.01 so that a decimal 100's chart is developed. Fractions can be created using today's date as a numerator, with a set denominator to use as a basis for discussions of equivalent fractions, improper fractions, and mixed numbers. Measurement can be discussed, using today's date as the basis for money, coins, number of inches, ounces, minutes, angles, etc. There are many possibilities for mathematizing every math domain by using the calendar as a generator of changing daily number (date or day of school year).

A monthly calendar board or corner can be set up with chart paper or dry erase paper to record predictions, vocabulary, and other elements. A Smartboard could also be used to organize calendar elements. If calendar elements are hung on the wall, then students will be able to refer to those visual displays as reminders and reinforcement at any time. The following elements should be in place in order to foster deep discussions and meaningful student engagement during calendar time:

- A safe environment
- Math models and tools, such as a hundreds chart, number line, measurement tools, play money
- Opportunities to think first and then discuss
- Student interaction and discourse

Number Talks

In order to be mathematically proficient, students must be able to compute accurately, efficiently, and flexibly. Daily classroom number talks provide a powerful avenue for developing "efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics." (Parrish, 2010) Number talks involve classroom conversations and discussions centered upon purposefully planned computation problems.

In Sherry Parrish's book, <u>Number Talks: Helping Children Build Mental Math and Computation Strategies</u>, teachers will find a wealth of information about Number Talks, including:

- Key components of Number Talks
- Establishing procedures
- Setting expectations
- Designing purposeful Number Talks
- Developing specific strategies through Number Talks

There are four overarching goals upon which K-2 teachers should focus during Number Talks. These goals are:

1. Developing number sense
2. Developing fluency with small numbers
3. Subitizing
4. Making Tens

Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. A great place to introduce a Number Talk is during Number Corner/Calendar Time. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent during the mathematics lesson**. In fact, teachers only need to spend 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. The primary goal of Number Talks is computational fluency. Children develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This, in turn, serves them well when they are asked to express their mathematical processes in writing. In order for children to become computationally fluent, they need to know particular mathematical concepts and strategies that go far beyond what is required to memorize basic facts or procedures.

Students will begin to understand major characteristics of numbers, such as:

- Numbers are composed of smaller numbers.
- Numbers can be taken apart and combined with other numbers to make new numbers.
- What we know about one number can help us figure out other numbers.
- What we know about parts of smaller numbers can help us with parts of larger numbers.
- Numbers are organized into groups of tens and ones (and hundreds, tens and ones and so forth).
- What we know about numbers to 10 helps us with numbers to 100 and beyond.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in many different ways: as dot cards, ten frames, sticks of cubes, models shown on the overhead, a word problem or a numerical expression. Strategies are *not explicitly taught* to students, instead the problems presented lead to various strategies.
2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a "thumbs-up" when they have determined their answer. The thumbs up signal is unobtrusive- a message to the teacher, not the other students.
3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.
4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.
5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."
6. **The steps are repeated for additional problems**.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of ways
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

For further guidance on Number Talks, please see the Effective Instructional Practices Guide: https://www.georgiastandards.org/Georgia-Standards/Documents/GSE-Effective-Instructional-Practices-Guide.pdf

Estimation 180

Estimation is a skill that has many applications, such as checking computation answers quickly. Engaging in regular estimation activities will develop students' reasoning skills, number sense, and increase their repertoire of flexible and efficient strategies. As students gain more experiences with estimation, their accuracy will improve.

According to John Van de Walle, there are three types of estimation that students should practice:

- Measurement estimation – determining an approximate measurement, such as weight, length, or capacity
- Quantity estimation – approximating the number of items in a collection
- Computational estimation – determining a number that is an approximation of a computation

One resource which provides contexts for all three types of estimation is Andrew Stadel's website, http://www.estimation180.com/. In his website, Mr. Stadel has posted daily estimation contexts. Here are his directions for using his website:

1. Click on a picture.
2. Read the question.
3. Look for context clues.
4. Make an estimate.
5. Tell us how confident you are.
6. Share your reasoning (what context clues did you use?).
7. See the answer.
8. See the estimates of others.

The most important part is step #6. After you make an estimate, feel free to give a brief description. It's so valuable to a classroom when students share their logic or use of context clues when formulating an estimate.

Andrew Stadel has collaborated with Michael Fenton to create a recording sheet for students to use with the estimation contexts on the website. The recording sheet can also be found at http://www.estimation180.com/. Here are his directions for the recording sheet:

Day #	Description	⬇ Too Low	Too High ⬆	My Estimate	My Reasoning	Answer	Error	|Error| as %
Ex. A	Tyler's age (months)	24	36	30	He looks a little older than my cousin (who is 2)	26	⊕ – 4	4/26 ≈ 15%
Ex. B	*Bohemian Rhapsody*	4:00	5:00	4:30	10% of song = 30 sec 300 sec total = 5 min	5:56	+ ⊖ 86	86/356 ≈ 24%
							+ –	

Column use descriptions from Andrew Stadel:

Day #
In Estimation 180's first year, I was just trying to keep up with creating these estimation challenges in time for use in my own classroom. There really wasn't a scope and sequence involved. That said, now that there are over 160 estimation challenges available, teachers and students can use them at any time throughout the school year and without completing them in sequential order. Therefore, use the *Day #* column simply to number your daily challenges according to the site. Tell your students or write it up on the board that you're doing the challenge from Day 135 even though you might be on the fifth day of school.

Description
In my opinion, this column is more important than the *Day #* column. Don't go crazy here. Keep it short and sweet, but as specific as possible. For example, there's a lot of scattered height estimates on the site. Don't write down "How tall?" for Day 110. Instead write "Bus height" because when you get to Day 111, I'd write in "Parking structure height". I believe the teacher has the ultimate say here, but it can be fun to poll your students for a short description in which you all can agree. Give students some ownership, right? If unit measurement is involved, try and sneak it in here. Take Day 125 for instance. I'd suggest entering "Net Wt. (oz.) of lg Hershey's bar." Keep in mind that Day 126 asks the same question, but I'd suggest you encourage your class to use pounds if they don't think of it.
*By the way, sometimes unit measurement(s) are already included in the question. Use discretion.

Too Low
Think of an estimate that is too low.
Don't accept one (1), that's just rubbish, unless one (1) is actually applicable to the context of the challenge. Stretch your students. Think of it more as an answer that's too low, but reasonably close. After all, this is a site of estimation challenges, not gimmees.

Too High
Refer to my notes in *Too Low*. Just don't accept 1 billion unless it's actually applicable. Discuss with students the importance of the *Too Low* and *Too High* sections: we are trying to eliminate wrong answers while creating a range of possible answers.

My Estimate
This is the place for students to fill in their answer. If the answer requires a unit of measurement, we better see one. Not every estimation challenge is "How many..." marshmallows? or Christmas lights? or cheese balls?
Even if a unit of measurement has already been established (see the *Description* notes), I'd still encourage your students to accompany their numerical estimate with a unit of measurement.

For example, on Day 41, "What's the height of the Giant [Ferris] Wheel?" use what makes sense to you, your students and your country's customary unit of measurement. Discuss the importance of unit measurements with students. Don't accept 108. What does that 108 represent? Pancakes? Oil spills? Bird droppings? NO! It represents 108 feet.

My Reasoning
The *My Reasoning* section is the most recent addition to the handout and I'm extremely thrilled about it. This is a student's chance to shine! Encourage their reasoning to be short and sweet. When a student writes something down, they'll be more inclined to share it or remember it. Accept bullet points or phrases due to the limited space. We don't need students to write paragraphs. However, we are looking for students to identify any context clues they used, personal experiences, and/or prior knowledge. Hold students accountable for their reasoning behind the estimate of the day.

Don't let student reasoning go untapped!
If you're doing a sequence of themed estimation challenges, don't accept, "I just guessed" after the first day in the sequence. For example, if you're doing the flight distance themed estimate challenges starting on Day 136, you will establish the distance across the USA on the first day. Sure, go ahead and guess on Day 136, but make sure you hold students accountable for their reasoning every day thereafter.
Have students share their reasoning before and after revealing the answer. Utilize Think-Pair-Share. This will help create some fun conversations *before* revealing the answer. After revealing the answer, get those who were extremely close (or correct) to share their reasoning. I bet you'll have some great mathematical discussions. I'm also curious to hear from those that are way off and how their reasoning could possibly be improved.

I'd say the *My Reasoning* section was born for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others.* Keep some of these thoughts in mind regarding Mathematical Practice 3:

- Explain and defend your estimate.
- Construct a detailed explanation referencing context clues, prior knowledge, or previous experiences.
- Invest some confidence in it.
- Try to initiate a playful and respectful argument in class.
- Ask "Was anyone convinced by this explanation? Why? Why not?" or "Are you guys going to let [student name] off the hook with that explanation?"

There's reasoning behind every estimate (not guess).

- Find out what that reasoning is!
- DON'T let student reasoning go untapped!

Answer

Jot down the revealed answer. I'd also encourage students to write down the unit of measurement used in the answer. The answer might use a different unit of measurement than what you and your class agreed upon. Take the necessary time to discuss the most relative unit of measurement. I might be subjectively wrong on some of the answers posted. As for more thoughts on unit of measurement, refer to the *My Estimate* notes above. Continue having mathematical discussion after revealing the answer. Refer to my notes regarding the use of Mathematical Practice 3 in the *My Reasoning* section.

Error

Find the difference between *My Estimate* and *Answer*. Have students circle either the "+" or the "-" if they didn't get it exactly correct.

+ Your estimate was **greater** than (above) the actual answer.

- Your estimate was **less** than (below) the actual answer.

Mathematize the World through Daily Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), and daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, and have productive discourse about the mathematics in which students are engaged. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of the routines is important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

Workstations and Learning Centers

When thinking about developing work stations and learning centers you want to base them on student readiness, interest, or learning profile such as learning style or multiple intelligence. This will allow different students to work on different tasks. Students should be able to complete the tasks within the stations or centers independently, with a partner or in a group.

It is important for students to be engaged in purposeful activities within the stations and centers. Therefore, you must careful consider the activities selected to be a part of the stations and centers. When selecting an activity, you may want to consider the following questions:

- Will the activity reinforce or extend a concept that's already been introduced?
- Are the directions clear and easy to follow?
- Are materials easy to locate and accessible?
- Can students complete this activity independently or with minimal help from the teacher?
- How will students keep a record of what they've completed?

- How will students be held accountable for their work?

(Laura Candler, *Teaching Resources*)

When implementing work stations and learning centers within your classroom, it is important to consider when the stations and centers will be used. Will you assign students to specific stations or centers to complete each week or will they be able to select a station or center of their choice? Will this opportunity be presented to all students during particular times of your math block or to students who finish their work early?

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned.

Games

“A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience.

Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used.” (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 28

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student’s thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? Assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

Representing

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, “We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.”

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. (Resource: Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*)

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from “doing it myself!”

Problem Solving Rubric (3-5)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, AND selected and applied an appropriate problem-solving strategy that led to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, AND selected and applied an appropriate problem-solving strategy that led to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem-solving strategy that led to a thorough and accurate solution.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using above-grade-level appropriate vocabulary in their process of finding solutions.
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tool or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition, this student was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

Many thanks to Richmond County Schools for sharing this rubric!

LITERATURE RESOURCES

- *Amanda Bean's Amazing Dream (A mathematical story)* by Cindy Neuschwander
- *Bigger, Better, Best!* by Stuart J. Murphy
- *Divide and Ride* by Stuart J. Murphy
- *Give Me Half!* By Stuart J. Murphy
- *Grandfather Tang's Story* by Ann Tompert
- *Grapes of Math* by Greg Tang
- *How Many Seeds in a Pumpkin?* by Margaret McNamara
- *Lemonade for Sale* by Stuart J. Murphy
- *One Hundred Hungry Ants*, by Elinor J. Pinczes
- *Picture Pie 2: A Drawing Book and Stencil* by Ed Emberely
- *Picture Pie: A Circle Drawing Book* by Ed Emberely
- *Racing Around* by Stuart J. Murphy
- *Spaghetti and Meatballs For All* by Marilyn Burns
- *The Best of Times* by Greg Tang
- *The Great Graph Contest* by Loreen Leedy
- *The Greedy Triangle* by Marilyn Burns
- *The Long Wait* by Annie Cobb
- *The Mathemagician's Apprentice* by Brian Boyd

TECHNOLOGY LINKS

Unit 1

- http://www.shodor.org/interactivate/activities/EstimatorFour/ A "Four in a Row" game where players get checkers when they quickly and efficiently estimate a sum to two numbers.
- Mental computation strategies with some fun graphics to demonstrate the strategies. - https://corrimal-p.schools.nsw.gov.au/content/dam/doe/sws/schools/c/corrimal-p/localcontent/maths_helpful_hints.pdf
- Graphs can be created using templates such as the pictograph template below: http://www.beaconlearningcenter.com/documents/2351_5255.pdf
- Pictographs can be created using excel following the directions below: http://faculty.kutztown.edu/schaeffe/Excel/Vallone/Vallone_Excel.pdf
- Bar graphs can be created using a website such as:
    - https://nces.ed.gov/nceskids/createagraph/ or
    - http://illuminations.nctm.org/ActivityDetail.aspx?ID=63
- If students are having difficulty thinking of a question, these websites have many ideas:
    0. http://www.canteach.ca/elementary/numbers13.html
    1. https://www.uen.org/lessonplan/download/14587?lessonId=10867&segmentTypeId=2
- This website allows students to create bar graphs based on random sets of shapes.
    2. http://www.shodor.org/interactivate/activities/BarGraphSorter/

Unit 2

- http://www.multiplication.com/games/all-games This site contains practice games for multiplication as well as teacher resource pages with instructional ideas on how to introduce multiplication.
- Making Arrays - http://education.abc.net.au/res/i/L2056/index.html This site allows students to make arrays and see the associated facts.
- Partial Product Finder - https://apps.mathlearningcenter.org/partial-product-finder/
- Array Factor Families http://education.abc.net.au/home#!/media/32921/arrays-factor-families This site allows students to connect arrays to multiplication and fact families.
- http://www.lessonplanspage.com/MathLAMultiplicationDivisionUsingTheDoorbellRang23.htm This website provides teacher resources for the book *The Doorbell Rang* by Pat Hutchins.
- http://www.softschools.com/math/games/division_practice.jsp This site allows for practice with division. The student or teacher can determine the parameters for the divisor, dividend, and number of problems
- https://apps.mathlearningcenter.org/pattern-shapes/ This site allows students to work with pattern blocks in an interactive applet and easily print their work.
- http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm#div This site for teachers and parents provides information on using base 10 blocks to solve division problems with an area model.
- http://www.eduplace.com/math/mw/background/3/08/te_3_08_overview.html This site provides background information on the relationship between multiplication and division.

Unit 3

- http://illuminations.nctm.org/ActivityDetail.aspx?id=46 This website provides actives for measuring the area of rectangles.
- http://www.mathplayground.com/area_perimeter.html
- http://www.scootle.edu.au/ec/viewing/L384/index.html
- https://educators.brainpop.com/bp-jr-topic/area/ Have students view the short video clip and discuss as needed.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=204 Students import their data into an online pictograph generator using the following website:
- http://www.youtube.com/watch?v=s2gSY1F5kQI Provides excellent review of how to collect, organize, and represent data to make a line plot based upon student results of rolling dice

Unit 4

- https://apps.mathlearningcenter.org/geoboard/ **Virtual Geoboard**
- http://www.mathcats.com/explore/polygons.html - Explore Polygons
- http://www.math-play.com/Polygon-Game.html - Name the Shape

- http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html - allows students to move corners to make sizes.
- http://www.interactivestuff.org/match/maker.phtml?featured=1&id=24 – matching game
- http://www.mathplayground.com/matching_shapes.html - matching games (includes kite)
- http://illuminations.nctm.org/LessonDetail.aspx?id=L350 Complete lesson on rectangles and parallelograms *(for members of NCTM)*
- http://illuminations.nctm.org/LessonDetail.aspx?id=L813 Shape Up Lesson from Illumination *(for members of NCTM)*
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L346 expanding pattern blocks *(for members of NCTM)*
- https://apps.mathlearningcenter.org/pattern-shapes/ **Pattern Block Fractions**
- http://people.rit.edu/mecsma/Professional/Puzzles/Pentominoes/P-A.html Provides several beginner problems with solutions for pentominoes.
- http://puzzler.sourceforge.net/docs/pentominoes.html Solutions to several pentomino puzzles such as the one below.
- http://highered.mcgraw-hill.com/sites/0072532947/student_view0/grid_and_dot_paper.html Various Grid and Dot Paper Templates
- http://www.shodor.org/interactivate/activities/ShapeExplorer/?version=1.6.0_07&browser=MSIE&vendor=Sun_Microsystems_Inc.&flash=10.0.32 Randomly generated rectangles for which the perimeter and the area can be found
- http://www.learner.org/courses/learningmath/measurement/session9/part_a/index.html An interactive task for ***teachers*** to explore area and perimeter.
- http://nces.ed.gov/nceskids/createagraph/ Create a bar graph online.
- http://www.softschools.com/math/data_analysis/pictograph/make_your_own_pictograph/ Create a Picture Graph

Unit 5

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=80
- http://www.dreambox.com/k-8-math-lessons (scroll to "**Placing Fractions on the Number Line")** This lesson engages students in actively placing fractions in their correct location on the number line. A number line representation ensures students understand how to compare and order fractions apart from any specific part-whole context. Instead of using a particular context, students use landmark fractions and numbers to place fractions on a number line from 0 to 1 and from 0 to 2.
- http://www.gamequarium.com/fractions.html
- http://www.gregtangmath.com/satisfraction *This game allows the user to filter by various comparison strategies (e.g., common numerator, common denominator, etc.) and requires students to vary between picking the largest and smallest fractions.
- http://www.kidsnumbers.com/turkey-terminator-math-game.php
- http://www.learningplanet.com/sam/ff/index.aspThis site has both teacher and student activities.
- http://www.mathplayground.com/Scale_Fractions.html
- http://www.mathsisfun.com/numbers/fraction-number-line.html

- http://www.mathsisfun.com/numbers/fractions-match-words-pizza.html
- http://www.sheppardsoftware.com/mathgames/fractions/AnimalRescueFractionsNumberLineGame.htm Students estimate position on an empty number line in these engaging games.
- http://www.visualfractions.com/sitemap.html
- http://www.visualfractions.com/FindGrampy/findgrampy.html
- https://www.brainpop.com/games/battleshipnumberline/
- https://www.conceptuamath.com/app/tool-library *Conceptua Learning Tools (Fraction Tab) are great for both parents and teachers while working on fraction concepts

Unit 6

- http://www.shodor.org/interactivate/activities/ElapsedTime/?version=disabled&browser=MSIE&vendor=na&flash=10.0.32 This website features several activities surrounding elapsed time using analog or digital clocks.
- http://donnayoung.org/math/clock.htm This website has printable blank clock faces
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=63 Students can use this website to enter their data and create a bar graph.
- http://www.shodor.org/interactivate/activities/BarGraph/
- https://www.mathplayground.com/balance_scales.html * Website that allows students to practice using a balance scale
- http://nces.ed.gov/nceskids/createagraph/default.aspx "Create A Graph" by the National Center for Education Statistics is a web-based program which allows students to create a bar graph.
- https://mathslinks.net/links/ambleside-grapher This site is limited in its use but very simple to use – most labels can be renamed.

RESOURCES CONSULTED

Content:
Mathematics Progressions Documents: http://ime.math.arizona.edu/progressions/

Illustrative Mathematics: https://www.illustrativemathematics.org/content-standards/3

Utah Education Network: https://www.uen.org/core/core.do?courseNum=5130

NZ Maths: http://nzmaths.co.nz/

Teacher/Student Sense-making:
http://www.youtube.com/user/mitcccnyorg?feature=watch
http://www.insidemathematics.org/index.php/video-tours-of-inside-mathematics/classroom-teachers/157-teachers-reflect-mathematics-teaching-practices
https://www.georgiastandards.org/Georgia-Standards/Pages/Math.aspx or
http://secc.sedl.org/common_core_videos/

<u>Journaling:</u>
http://www.mathsolutions.com/index.cfm?page=wp10&crid=3

<u>Additional Resources</u>:
https://www.kentuckymathematics.org/pimser.php

<u>Community of Learners:</u>
http://www.edutopia.org/math-social-activity-cooperative-learning-video
http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos
http://www.responsiveclassroom.org/category/category/first-weeks-school